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Dominique Chapelle · Klaus-Jürgen Bathe

The Finite Element Analysis of Shells – Fundamentals

Second Edition

 Springer

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To our fruitful collaboration and long-lasting friendship

Preface – Second Edition

Since our writing of the first edition of this book, some important advances in finite element methods for shell analyses have been achieved. In this new edition, we present some of these new developments. In addition, we have endeavored to strengthen the presentation throughout the text.

The major changes and additions in the book pertain to the section on the presentation of general variational formulations and finite element approximations, the new sections on 3D-shell models and finite element discretizations, on dynamic analyses, and on triangular elements. We also added more explanations in the text, and many new examples and figures in order to render the book more complete.

We endeavored to achieve more completeness also because the analysis of shells represents one of the most challenging fields in *all* of mechanics, and encompasses various fundamental and generally applicable components. Specifically, the material presented in this book regarding geometric descriptions, tensors and mixed variational formulations is fundamental and applicable also in other areas of mechanics.

However, in the writing of this second edition, we have not changed our basic objective, that is, in the book we continue to focus on the fundamentals of shell analysis and the synergy between the physical and mathematical understanding – as we have described in more detail in the Preface of the first edition.

Finally, in addition to the acknowledgments made in the first edition, Dominique Chapelle would like to warmly thank his colleague and friend Philippe Moireau for his feedback on this second edition, and Klaus-Jürgen Bathe would like to acknowledge his very valuable collaboration on advancing methods for shell analysis, reflected by references and results in this book, with his former student and now a Professor at KAIST, Phill-Seung Lee.

D. Chapelle and K.J. Bathe

Preface – First Edition

Since the first developments of finite element methods for the analysis of shells, about half a century ago, the possibilities to analyze shells in designs and to study, in general, the behavior of shell structures have vastly increased. However, at the same time as shell finite element procedures were rapidly introduced in many everyday practices of engineering analysis, also, a large research effort was directed towards increasing the capabilities of shell finite element methods, for linear and nonlinear analyses. This research effort is still ongoing because of the great challenges in shell structural analyses. These challenges are largely due to the diversity of shell structural behaviors and the difficulties to solve for such behaviors in a reliable and uniformly effective, ideally optimal, manner. The difficulties are apparent when considering complex shells of arbitrary curvatures, material conditions, boundary supports, loading, and in particular of small thickness. Furthermore, these challenges have grown during the last decades, and probably will continue to grow, because increasingly more daring – but also more beautiful – shell structures have been designed and analyzed than previously thought possible.

The earliest studies of shell structures using analytical methods were performed well over a century ago. At that time, and indeed until the development of the finite element method, researchers used all the available *physical and mathematical understanding* to formulate shell theories and solve shell problems. While the shell theories were quite general in nature, the actual solutions obtained to a shell problem were mostly very approximate, that is, when compared with the actual physical behavior of the shell considered.

With the development of the modern finite element method, the approach towards the practical solution of shell problems changed. Finite element procedures were largely developed based on physical understanding without the use of mathematical shell theories. Indeed, various shell finite element methods were proposed by simply superimposing plate bending and plane stress membrane behaviors. With this approach many shell structures were successfully analyzed, but of course only to a certain level of accuracy. In fact, the significant limitations of such element formulations became also apparent and, to some extent, recourse to the use of shell theories was sought to develop more powerful finite element methods.

The effective study of shell structures clearly requires a deep physical understanding of shell structural behaviors. The development of more powerful finite element methods requires in addition a strong knowledge of mathematical theories. Indeed, it is clearly the *synergy between physical and mathematical understanding* that will advance our knowledge of shell structural behaviors and the development of finite element methods. This, in fact, corresponds to the approach taken many years ago in the study of shell structures, but is now a path more difficult to follow. Namely, the mathematical theories now available are much more advanced and are still developing.

Our objective with this book is to provide a text in which the fundamentals of shell theories, mathematical analyses, and finite element methods are presented with a view towards the synergy mentioned above. Hence we endeavor to present and emphasize the physical concepts of shell analyses and their mathematical bases. With the text directed towards finite element solutions, we explore the complexity of the physical behaviors of shells in order to explain the difficulties to obtain uniformly optimal finite element procedures. While, of course, already quite effective finite element methods are available, our aim is, as well, to provide important fundamental considerations – including basic testing procedures – for the development of more powerful methods.

An important aspect of a shell finite element method is that it be reliable. We discuss this aspect in depth and direct our attention only towards the development of such finite element methods. Only finite element analysis procedures that are reliable and effective can be employed confidently and will be used for many years to come. With such shell analysis techniques, engineers can confidently replace some expensive laboratory testing and, using finite element analysis, experiment at much less expense with exciting new design concepts. In this way, the full benefits of computer-aided engineering are attained.

This text is not intended to be a survey of finite element methods for shell analyses. Indeed, it is a text in which we – to a large extent – only discuss our experiences with shell analysis methods with the focus on the endeavors mentioned above.

The writing of this book required a large effort and we would like to thank all those who have supported us, and made it possible for us, to complete this work.

Dominique Chapelle is grateful to his colleagues at INRIA-Rocquencourt – and in particular to his colleagues in the MACS project – for providing him with a very stimulating and pleasant environment. He would like to extend his special thanks to Marina Vidrascu for her precious help in obtaining some of the numerical results presented in the book.

Klaus-Jürgen Bathe is grateful to the Department of Mechanical Engineering, M.I.T., for the excellent environment made available for his teaching, research and scholarly writing, and to the many students who have con-

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We both would like to thank Michel Bernadou, Carlo Lovadina and Olivier Pironneau for their valuable comments regarding this book.

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D. Chapelle and K.J. Bathe

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