Fluid Mechanics for Engineers

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A Graduate Textbook

Springer

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Preface

The contents of this book covers the material required in the Fluid Mechanics Graduate Core Course (MEEN-621) and in Advanced Fluid Mechanics, a Ph.D-level elective course (MEEN-622), both of which I have been teaching at Texas A&M University for the past two decades. While there are numerous undergraduate fluid mechanics texts on the market for engineering students and instructors to choose from, there are only limited texts that comprehensively address the particular needs of graduate engineering fluid mechanics courses. To complement the lecture materials, the instructors more often recommend several texts, each of which treats special topics of fluid mechanics. This circumstance and the need to have a textbook that covers the materials needed in the above courses gave the impetus to provide the graduate engineering community with a coherent textbook that comprehensively addresses their needs for an advanced fluid mechanics text. Although this text book is primarily aimed at mechanical engineering students, it is equally suitable for aerospace engineering, civil engineering, other engineering disciplines, and especially those practicing professionals who perform CFD-simulation on a routine basis and would like to know more about the underlying physics of the commercial codes they use. Furthermore, it is suitable for self study, provided that the reader has a sufficient knowledge of calculus and differential equations.

In the past, because of the lack of advanced computational capability, the subject of fluid mechanics was artificially subdivided into inviscid, viscous (laminar, turbulent), incompressible, compressible, subsonic, supersonic and hypersonic flows. With today's state of computation, there is no need for this subdivision. The motion of a fluid is accurately described by the Navier-Stokes equations. These equations require modeling of the relationship between the stress and deformation tensor for linear and nonlinear fluids only. Efforts by many researchers around the globe are aimed at directly solving the Navier-Stokes equations (DNS) without introducing the Reynolds stress tensor, which is the result of an artificial decomposition of the velocity field into a mean and fluctuating part. The use of DNS for engineering applications seems to be out of reach because the computation time and resources required to perform a DNS-calculation are excessive at this time. Considering this constraining circumstance, engineers have to resort to Navier-Stokes solvers that are based on Reynolds decomposition. It requires modeling of the transition process and the Reynolds stress tensor to which three chapters of this book are dedicated.

The book is structured in such a way that all conservation laws, their derivatives and related equations are written in coordinate invariant forms. This type of structure enables the reader to use Cartesian, orthogonal curvilinear, or non-orthogonal body fitted coordinate systems. The coordinate invariant equations are then decomposed into components by utilizing the index notation of the corresponding coordinate systems. The use of a coordinate invariant form is particularly essential in understanding the underlying physics of the turbulence, its implementation into the Navier-Stokes equations, and the necessary mathematical manipulations to arrive at different correlations. The resulting correlations are the basis for the following turbulence modeling. It is worth noting that in standard textbooks of turbulence, index notations are used throughout with almost no explanation of how they were brought about. This circumstance adds to the difficulty in understanding the nature of turbulence by readers who are freshly exposed to the problematics of turbulence. Introducing the coordinate invariant approach makes it easier for the reader to follow step-by-step mathematical manipulations, arrive at the index notation and the component decomposition. This, however, requires the knowledge of tensor analysis. Chapter 2 gives a concise overview of the tensor analysis essential for describing the conservation laws in coordinate invariant form, how to accomplish the index notation, and the component decomposition into different coordinate systems.

Using the tensor analytical knowledge gained from Chapter 2, it is rigorously applied to the following chapters. In Chapter 3, that deals with the kinematics of flow motion, the Jacobian transformation describes in detail how a time dependent volume integral is treated. In Chapter 4 and 5 conservation laws of fluid mechanics and thermodynamics are treated in differential and integral forms. These chapters are the basis for what follows in Chapters 7, 8, 9, 10 and 11 which exclusively deal with viscous flows. Before discussing the latter, the special case of inviscid flows is presented where the order of magnitude of a viscosity force compared with the convective forces are neglected. The potential flow, a special case of inviscid flow characterized by zero vorticity $\nabla \times \boldsymbol{V} = \mathbf{0}$, exhibited a major topic in fluid mechanics in pre-CFD era. In recent years, however, its relevance has been diminished. Despite this fact, I presented it in this book for two reasons. (1) Despite its major short comings to describe the flow pattern directly close to the surface, because it does not satisfy the no-slip condition, it reflects a reasonably good picture of the flow outside the boundary layer. (2) Combined with the boundary layer calculation procedure, it helps acquiring a reasonably accurate picture of the flow field outside and inside the boundary layer. This, of course, is valid as long as the boundary layer is not separated. For calculating the potential flows, conformal transformation is used where the necessary basics are presented in Chapter 6, which is concluded by discussing different vorticity theorems.

Particular issues of laminar flow at different pressure gradients associated with the flow separation in conjunction with the wall curvature constitute the content of Chapter 7 which seamlessly merges into Chapter 8 that starts with the stability of laminar, followed by laminar-turbulent transition, intermittency function and its implementation into Navier-Stokes. Averaging the Navier-Stokes equation that includes the intermittency function leading to the Reynolds averaged Navier-Stokes equation (RANS), concludes Chapter 8. In discussing the RANS-equations, two quantities have to be accurately modeled. One is the intermittency function, and the other is the Reynolds stress tensor with its nine components. Inaccurate modeling of these two quantities leads to a multiplicative error of their product. The transition was already discussed in Chapter 8 but the Reynolds stress tensor remains to be modeled.

This, however, requires the knowledge and understanding of turbulence before attempts are made to model it. In Chapter 9, I tried to present the quintessence of turbulence required for a graduate level mechanical engineering course and to critically discuss several different models. While Chapter 9 predominantly deals with the wall turbulence, Chapter 10 treats different aspects of free turbulent flows and their general relevance in engineering. Among different free turbulent flows, the process of development and decay of wakes under positive, zero, and negative pressure gradients is of particular engineering relevance. With the aid of the characteristics developed in Chapter 10, this process of wake development and decay can be described accurately.

Chapter 11 is entirely dedicated to the physics of laminar, transitional and turbulent boundary layers. This topic has been of particular relevance to the engineering community. It is treated in integral and differential forms and applied to laminar, transitional, turbulent boundary layers, and heat transfer.

Chapter 12 deals with the compressible flow. At first glance, this topic seems to be dissonant with the rest of the book. Despite this, I decided to integrate it into this book for two reasons: (1) Due to a complete change of the flow pattern from subsonic to supersonic, associated with a system of oblique shocks makes it imperative to present this topic in an advanced engineering fluid text; (2) Unsteady compressible flow with moving shockwaves occurs frequently in many engines such as transonic turbines and compressors, operating in off-design and even design conditions. A simple example is the shock tube, where the shock front hits the one end of the tube to be reflected to the other end. A set of steady state conservation laws does not describe this unsteady phenomenon. An entire set of unsteady differential equations must be called upon which is presented in Chapter 12. Arriving at this point, the students need to know the basics of gas dynamics. I had two options, either refer the reader to existing gas dynamics textbooks, or present a concise account of what is most essential in following this chapter. I decided on the second option.

At the end of each chapter, there is a section that entails problems and projects. In selecting the problems, I carefully selected those from the book Fluid Mechanics Problems and Solutions by Professor Spurk of Technische Universität Darmstadt which I translated in 1997. This book contains a number of highly advanced problems followed by very detailed solutions. I strongly recommend this book to those instructors who are in charge of teaching graduate fluid mechanics as a source of advanced problems. My sincere thanks go to Professor Spurk, my former Co-Advisor, for giving me the permission . Besides the problems, a number of demanding projects are presented that are aimed at getting the readers involved in solving CFD-type of problems. In the course of teaching the advanced Fluid Mechanics course MEEN-622, I insist that the students present the project solution in the form of a technical paper in the format required by ASME Transactions, Journal of Fluid Engineering.

In typing several thousand equations, errors may occur. I tried hard to eliminate typing, spelling and other errors, but I have no doubt that some remain to be found by readers. In this case, I sincerely appreciate the reader notifying me of any mistakes found; the electronic address is given below. I also welcome any comments or suggestions regarding the improvement of future editions of the book.

My sincere thanks are due to many fine individuals and institutions. First and foremost, I would like to thank the faculty of the Technische Universität Darmstadt from whom I received my entire engineering education. I finalized major chapters of the manuscript during my sabbatical in Germany where I received the Alexander von Humboldt Prize. I am indebted to the Alexander von Humboldt Foundation for this Prize and the material support for my research sabbatical in Germany. My thanks are extended to Professor Bernd Stoffel, Professor Ditmar Hennecke, and Dipl. Ing. Bernd Matyschok for providing me with a very congenial working environment.

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Last, but not least, my special thanks go to my family, Susan and Wilfried for their support throughout this endeavor.

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Contents

Nomenclature

A	acceleration vector
b	wake width
c	complex eigenfunction, $c = c_r + ic_i$
c	speed of sound
c_p, c_v	specific heat capacities
C	von Kármán constant
C_D	drag coefficient
C_f	friction coefficient
C_p	pressure coefficient
D	deformation tensor
D	total differential operator in absolute frame of reference
D	van Driest's damping function
D_R	total differential operator in relative frame of reference
e	specific total energy
e_i	orthonormal unit vector
E	Source (+), sink (-) strength
E	total energy
E(k)	energy spectrum
f_S	sampling frequency
F	force
F(z)	complex function
g_i, g^i	co-, contravariant base vectors in orthogonal coordinate system
g_{ij}, g^{ij}	co-, contravariant metric coefficients
$\mathbf{G}_i$	transformation vector
h, H	specific static, total enthalpy
H_{12}	boundary layer momentum form factor, $H_{12} = \delta_1/\delta_2$
H_{13}	boundary layer energy form factor, $H_{32} = \delta_3/\delta_2$
$\dot{q}$	heat flux
I(**x**,t)	intermittency function
I_1, I_2, I_3	principle invariants of deformation tensor
J	Jacobian transformation function
k	thermal conductivity
k	wave number vector
K	specific kinetic energy
l_m	Prandtl mixing length
$L_{ij}(\boldsymbol{x},t)$	turbulence length scale
m	mass

$\dot{m}$	mass flow
M	Mach number
$\mathbf{M}$	vector of moment of momentum
$\mathbf{M}_a$	axial vector of moment of momentum
$\mathbf{n}$	normal unit vector
N	Navier-Stokes operator
Nu	Nusselt number
p	static pressure
$\tilde{p}$	deterministic pressure fluctuation
p^+	dimensionless pressure gradient
p'	random pressure fluctuation
P, p_o	total (stagnation) pressure, $P = p + \rho V^2/2$
Pr	Prandtl number
Pr_e	effective Prandtl number
Pr_t	turbulent Prandtl number
q	specific thermal energy
Q	thermal energy
$\dot{\boldsymbol{q}}$	heat flux vector
R	radius in conformal transformation
Re	Reynolds number
Re_{crit}	critical Re
$\boldsymbol{R}(\boldsymbol{x},t,\boldsymbol{r},\tau)$	correlation second order tensor
s	specific entropy
St	Stanton number
Str	Strouhal number
S, S(t)	fixed, time dependent surface
t	time
$\mathbf{t}$	tangential unit vector
$T_{ij}(\boldsymbol{x},t)$	turbulence time scale
T	static temperature
$\mathbf{T}$	stress tensor, $\mathbf{T} = e_i e_j \tau_{ij}$
T_o	stagnation or total temperature
Tr	trace of second order tensor
T_n (y)	Chebyshev polynomial of first kind
u	specific internal energy
u	velocity
u_τ	wall friction velocity
u^+	dimensionless wall velocity, $u^+ = u/u_\tau$
U	undisturbed potential velocity
$\mathbf{U}$	rotational velocity vector
$\overline{U_1}$	time averaged wake velocity defect
$\overline{U_I}$	time averaged wake momentum defect

$\overline{U_{1m}}$	maximum velocity defect
v	specific volume
V	volume
V_0	fixed volume
V(t)	time dependent volume
V	absolute velocity vector
$\mathbf{V}_L$	velocity vector, laminar solution
$\mathbf{V}_T$	velocity vector, turbulent solution
$\tilde{\boldsymbol{V}}$	deterministic velocity fluctuation vector
$\bar{\boldsymbol{V}}$	mean velocity vector
$\boldsymbol{V}'$	random velocity fluctuating vector
V_i, V^j	co- and contravariant component of a velocity vector
$<\boldsymbol{V}>$	ensemble averaged velocity vector
w_m	specific shaft power
W	mechanical energy
$\dot{W}$	mechanical energy flow (power)
$\dot{W}_{sh}$	shaft power
W	relative velocity vectors
x_i	coordinates
y^+	dimensionless wall distance, $y^+ = u_\tau y/\nu$
z	complex variable

Greek Symbols

α	heat transfer coefficient
α	real quantity in disturbance stream function
β_i	disturbance amplification factor
β_r	circular disturbance frequency
$\gamma(x)$	time averaged intermittency factor, $\gamma(x) = \bar{I}$
$<\gamma(t)>$	ensemble averaged intermittency at a fixed position
$<\gamma(t)>_{max}$	ensemble averaged maximum intermittency at a fixed position
$<\gamma(t)>_{min}$	ensemble averaged minimum intermittency at a fixed position
Γ	circulation strength
Γ	relative intermittency
$\boldsymbol{\Gamma}$	circulation vector
Γ^i_{jk}	Christoffel symbol
γ_{min}, γ_{max},	minimum, maximum intermittency
δ	Kronecker delta
δ_1, δ_2, δ_3,	boundary layer displacement, momentum, energy thickness

ε	turbulence dissipation
ε_h	eddy diffusivity
ε_m	eddy viscosity
ε_{ijk}	permutation symbol
ζ	dimensionless periodic parameter
ζ	Kolmogorov's length scale
ζ	total pressure loss coefficient
Θ	shock expansion angle
$\Theta_{ij}(k_1,t)$	one-dimensional spectral function
κ	isentropic exponent,
κ	ratio of specific heats
κ	von Kármán constant
λ	disturbance wave length
λ	eigenvalue
λ	Taylor micro length scale
$\boldsymbol{\lambda}$	tangent unit vector
μ	absolute viscosity
μ	Mach angle
ν	expansion angle
ν	kinematic viscosity
ξ	dimensionless coordinate, $\xi = x/L$
$\boldsymbol{\xi}$	position vector in material coordinate system
η	dimensionless coordinate, $\eta = y/L$
η	Kolmogorov's length scale
π	pressure ratio
$\boldsymbol{\Pi}$	stress tensor, $\boldsymbol{\Pi} = e_i e_j \pi$
ρ	density
$\rho_{ij}(\boldsymbol{x},t,\boldsymbol{r},\tau)$	dimensionless correlation coefficient
τ	Kolmogorov's time scale
τ_o, τ_W	wall sear stress
υ	Kolmogorov's velocity scale
φ_1	dimensionless wake velocity defect $\varphi_1 = \bar{U}_1/\bar{U}_{1m}$
Φ	dissipation function
Φ, ψ	potential, stream function
$\boldsymbol{\Phi}(\boldsymbol{k},t)$	spectral tensor
Ψ	mass flow function
X	complex function
ω	angular velocity
$\boldsymbol{\omega}$	vorticity vector
$\boldsymbol{\Omega}$	Rotation tensor

Subscripts, Superscripts

∞	freestream
a, t	axial, tangential
ex	exit
in	inlet
max	maximum
min	minimum
s	isentropic
t	turbulent
w	wall
$-$	time averaged
$'$	random fluctuation
$\sim$	deterministic fluctuation
*	dimensionless
+	wall functions