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Christian Caron
Springer Heidelberg
Physics Editorial Department I
Tiergartenstrasse 17
69121 Heidelberg/Germany
christian.caron@springer.com

Daniel C. Cabra · Andreas Honecker
Pierre Pujol
Editors

Modern Theories of Many-Particle Systems in Condensed Matter Physics

 Springer

Prof. Daniel C. Cabra
Departamento de Física/IFLP
Universidad Nacional de La Plata
C.C. 67 La Plata, Argentina
e-mail: cabra@fisica.unlp.edu.ar

Privatdozent Dr. Andreas Honecker
Institut für Theoretische Physik
Georg-August-Universität Göttingen
Friedrich-Hund-Platz 1
37077 Göttingen
Germany
e-mail: ahoneck@uni-goettingen.de

Prof. Pierre Pujol
IRSAMC, Laboratoire de Physique
Théorique
Université Paul Sabatier
Route de Narbonne 118
Toulouse Cedex 4, 31062
France
e-mail: pierre.pujol@irsamc.ups-tlse.fr

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Preface

The discovery in 1986 of superconductors with a high critical temperature has initiated a dramatic renewal in modern condensed-matter physics. The richness of the phase diagram in such materials illustrates the complexity in the underlying physics. One key ingredient behind such a complex behavior is the interaction between particles, as many phases observed in experiments cannot be explained within a context of conventional solid-state physics based on a single-particle picture. This simple observation is the core of the subject of strongly correlated systems.

Systems where interactions are strong and play a crucial role are inherently difficult to analyze theoretically. The situation is particularly interesting in low-dimensional systems, where on the one hand quantum fluctuations and interactions play a crucial role, and on the other hand many sophisticated techniques have been developed. More precisely, from an analytical perspective, the development of non-perturbative methods and the study of integrable field theory have facilitated the understanding of the behavior of many quasi one-dimensional strongly correlated systems. This progress on analytical techniques was accompanied by a comparable advance in the development of numerical techniques.

From an experimental point of view, the study of modern condensed-matter physics has also been enriched by the development of devices of sizes in the nanometer region. Furthermore, cold-atom and novel solid-state systems such as graphene have emerged as new testing grounds for theoretical ideas. Both analytical and numerical techniques have been adapted accordingly such that the current understanding of condensed matter systems differs considerably from textbooks.

The aim of the 2009 Les Houches school on “Modern theories of correlated electron systems” was to provide an overview about recent developments in the theory of strongly correlated electrons and related problems. On the occasion of this school, it was generally felt that it would be useful to edit the Lecture Notes as a book and you are holding the result in your hands.

Nevertheless, in order to be of lasting value, some adjustments were made concerning the content of this book as compared to the 2009 school. In particular,

this book only has eight chapters as compared to the 12 series of lectures in the school.

The lectures of Subir Sachdev led to a general introduction to quantum phase transitions of antiferromagnets and the cuprate-based high-temperature superconductors. Eduardo Fradkin's lectures and the corresponding chapter on electronic liquid crystal phases in strongly correlated systems may appear a more specialized topic, but still constitute a unique overview of this subject. Antonio Castro Neto covered selected topics in graphene physics. Even if this is just one material, this has become one of the biggest efforts in contemporary Condensed Matter Physics (which was even honored with the Nobel Prize in Physics 2010). Accordingly, it is very difficult if not impossible to provide a complete overview of the field, but this chapter covers the basic quantum chemistry and elastic properties of a sheet of graphene in a manner which is complementary to other reviews.

In the school there were two lectures by Antoine Georges and Alexander Lichtenstein on Dynamical Mean-Field Theory and applications to correlated materials. These two authors joined forces with Hartmut Hafermann, Frank Lechermann, Alexei N. Rubtsov, and Mikhail I. Katsnelson to write one chapter on this topic. Two further lectures were given by Alexander Altland and Reinhold Egger on disordered electronic systems and transport through quantum dots. There is one joint chapter in this volume by these two authors which focuses on the second topic.

Further lectures in the school and the corresponding three final chapters of this volume make contact with related fields. Maciej Lewenstein gave a quantum information perspective on many-body physics; the corresponding chapter was written with the help of Remigiusz Augusiak and Fernando Cucchietti. Roderich Moessner covered the wide field of frustrated magnetism in his lectures; Chris Laumann, Antonello Scardicchio, and Shivaji Sondhi contributed to the resulting chapter on the statistical mechanics of classical and quantum computational complexity which focuses on a specific new development in this field. The lectures of Giuseppe Mussardo covered integrable methods in statistical field theory and the corresponding chapter concludes this volume.

Last but not least, we would like to thank the contributing authors for the effort which they have put in the individual chapters as well as the INSTANS network of the European Science Foundation, the Deutsch-Französische Hochschule/Université franco-allemande, and the Centre national de la recherche scientifique (CNRS) for the financial support of the 2009 school which ultimately made this volume possible.

Les Houches, April 2011

Daniel C. Cabra
Andreas Honecker
Pierre Pujol



May 27, 2009

Andreas Honecker Pierre Pujol

Alexander Altland
Eduardo Fradkin



Giuseppe Mussardo
Daniel Cabra



Antonio Castro Neto



Subir Sachdev



Maciej Lewenstein



Alexander Lichtenstein



Antoine Georges

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