

# Foundations of Engineering Mechanics

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*Series Editors: V.I. Babitsky, J. Wittenburg*



Liudmila Ya. Banakh · Mark L. Kempner

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# Vibrations of Mechanical Systems with Regular Structure

With 109 Figures

 Springer

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# Preface

In this book, regular structures are defined as periodic structures consisting of repeated elements (translational symmetry) as well as structures with a geometric symmetry. Regular structures have for a long time been attracting the attention of scientists by the extraordinary beauty of their forms. They have been studied in many areas of science: chemistry, physics, biology, etc. Systems with geometric symmetry are used widely in many areas of engineering. The various kinds of bases under machines, cyclically repeated forms of stators, reduction gears, rotors with blades mounted on them, etc. represent regular structures.

The study of real-life engineering structures faces considerable difficulties because they comprise a great number of working mechanisms that, in turn, consist of many different elastic subsystems and elements. The computational models of such systems represent a hierarchical structure and contain hundreds and thousands of parameters. The main problems in the analysis of such systems are the dimension reduction of model and revealing the dominant parameters that determine its dynamics and form its energy nucleus.

The two most widely used approaches to the simulation of such systems are as follows:

1. Methods using lumped parameters models, i.e., a discretization of the original system and its representation as a system with lumped parameters [including finite-element method (FEM)].
2. The use of idealized elements with distributed parameters and known analytical solutions for both the local elements and the subsystems.

Each of these approaches has its own specific characteristics and methods of study, which are described in [Chap. 2](#). On this basis, the book contains two parts.

The first part is devoted to the study of vibrations in linear systems with lumped (concentrated) parameters. In this part, dispersion equations (for systems with a periodic structure) as well as the theory of groups representation (for systems with geometric symmetry) are used. However, the specifics of the mechanical systems required a certain generalization of these approaches: Generalized projective operators of symmetry were introduced. This made it possible to take into account the

symmetry hierarchy in system, the small asymmetry arising as a result of the technological errors. The interactions of natural modes at vibroisolation of a body mounted on a symmetric frame and also the dynamic properties of a planetary reduction gear were investigated. The aforementioned approaches permit one to carry out the significant numerical simplification, using the characteristics of only one cell, and by this way to obtain analytical results.

The second part is devoted to systems with distributed parameters. In this part, the methods of dynamic compliances (stiffness) proposed by Professor M. L. Kempner are developed. In these approaches as form-building cells the idealized elements with distributed parameters are used. Methods for calculating such systems have been worked out. The basic matrices describing these systems, such as the transition matrix and the mixed matrix, as well as equations in finite differences, have been determined. With the help of these approaches, it is possible to obtain results in analytical form even for such complex elements of mechanical engineering structures as turbine disks with a blades package, multimass rotors, and cylindrical shells with circular and longitudinal ribbing. Both symmetric elements and reflection symmetry elements were treated.

Self-similar structures (fractals), in which every consecutive element is formed from the preceding one by a rotation and/or similarity transformation (scaling), can also be added to the regular structures. Examples of such systems in mechanical engineering are crankshafts, rods and beams whose parameters change in the same ratio from section to section, and the rotors of compressors where the disks are connected by means of conic shells.

The dynamic properties of such mechanical structures have not been studied thoroughly enough until now. A number of their dynamic properties are provided in this book. In particular, it is shown that some classes of self-similar structures represent a mechanical band filter. The natural frequencies of such systems as elements of flying machines e.g. rotors with self-similar disks and rotors of the drum type have been determined.

The book contains many numerical examples for real-life engineering structures, in particular aviation engines.

Part I was written by L.Y. Banakh, Part II by M.L. Kempner, and Sect. 3.4 by P.S. Akhmetkhanov.

This book is based on the original research the authors. The authors would like to thank Prof. V. Babitsky for his attention to this work.

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