

Lecture Notes in Mathematics

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Preface

The Lecture Notes collect seven mini-courses presented at the 5th Prague Summer School on Mathematical Statistical Physics that took place during two weeks of September 2006. As with preceding schools, it was aimed at PhD students and young postdocs. The central theme of the volume is what could be called “mathematics of phase transitions” in diverse contexts. Even though all courses were meant to introduce the reader to recent progress of a particular topic of modern statistical physics, attention has been paid to providing a solid grounding by carefully developing various basic tools.

One of the techniques that led, more than two decades ago, to a series of important outcomes in the theory of phase transitions of lattice models was reflection positivity. Recently it resurfaced and was used to obtain interesting new results in various settings. The lectures of Marek Biskup include a thorough introduction to reflection positivity as well as a review of its recent applications. In addition, it contains a crash course on lattice spin models that is useful as a background for other lectures of the collection.

Also the following two contributions concern equilibrium statistical physics. The lectures of Dmitri Ioffe are devoted to a stochastic geometric reformulation of classical as well as quantum Ising models. A unified approach to the Fortuin-Kasteleyn and random current representations in terms of path integrals is presented.

Statistical mechanics of directed polymers interacting with one-dimensional spatial effects is a topic with various applications in physics and biophysics. The lectures of Fabio Toninelli are devoted to a thorough discussion of the localization/delocalization transition in these models.

Metastability is a topic that has attracted a lot of attention recently. Here it is discussed in the notes of Anton Bovier and Frank den Hollander. The emphasis of the course of Anton Bovier is on a general rigorous framework. It explores how distinct time scales arise in Markov processes and how the metastable exit times can be expressed in terms of capacity, the crucial notion coming from potential theory. The lectures by Frank den Hollander are then devoted to a nontrivial application to metastability in the context of Glauber and Kawasaki dynamics of lattice gases. The main step is the careful evaluation of the relevant capacity in these particular cases.

Readers can have a glimpse of the prolifically developing nonequilibrium realm in the remaining two contributions. The lectures that were presented by Christian Maes and Karel Netočný form a pedagogical account of several recently discussed topics, with an emphasis on general principles.

Facilitated spin models, also known as kinetically constrained spin models, are reflecting important peculiar features of glassy dynamics. The lectures of Fabio Martinelli, submitted here with his coauthors, review mathematical results that contributed to a settlement of questions arising from numerical simulations.

Only one mini-course presented in Prague was not included into the present volume. These are the lectures about computational complexity and phase transitions in combinatorial optimisation presented by Stefan Mertens. The main reason for this omission is that his presentation was based on the recent monograph written by him and Cris Moore that already covers very well this topic.

The School was organised by Center for Theoretical Study (through the grant MSM 0021620845) with the Institute of Theoretical Computer Science at Charles University providing their beautiful lecture room in the historical centre. It could not have happened without the support of the European Science Foundation under the auspices of the programme Phase Transitions and Fluctuation Phenomena for Random Dynamics in Spatially Extended Systems. But most of all, the success of the School was determined by the lecturers as well as the students who created a pleasant and stimulating atmosphere. We hope that this spirit found its way into the written version of the lecture notes and will be appreciated by the reader.

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