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Fuzziness and Approximate Reasoning

Epistemics on Uncertainty, Expectation and Risk in Rational Behavior

Springer

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To Kenneth Arrow, Herbert Simon, Jacob Marschak, David Blackwell, Maurice Allais, Leonard Savage and Bruno de Finetti;

To all dedicated researchers working on stochastic uncertainty and risk;

To all dedicated researchers working on fuzzy uncertainty and risk;

To all dedicated researchers working to unify both: Stochastic uncertainty and risk on one hand, and fuzzy uncertainty and risk on the other;

To the initial members of North American Fuzzy Information Processing Group, may your works yield useful contributions to our knowledge on the phenomena of uncertainty, expectations and risk in the understanding the role of fuzziness in Decision-Choice Rationality.

To the Creative Force, the Spirit Force and the Light Force: Three in One, and One in Three.

Preface

We do not perceive the present as it is and in totality, nor do we infer the future from the present with any high degree of dependability, nor yet do we accurately know the consequences of our own actions. In addition, there is a fourth source of error to be taken into account, for we do not execute actions in the precise form in which they are imaged and willed.

Frank H. Knight [R4.34, p. 202]

The "degree" of certainty of confidence felt in the conclusion after it is reached cannot be ignored, for it is of the greatest practical significance. The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon favorableness of the opinion itself. The ultimate logic, or psychology, of these deliberations is obscure, a part of the scientifically unfathomable mystery of life and mind.

Frank H. Knight [R4.34, p. 226-227]

With some inaccuracy, description of uncertain consequences can be classified into two categories, those which use exclusively the language of probability distributions and those which call for some other principle, either to replace or supplement.

Kenneth Arrow [R21.5, p.8]

The basic need for a special theory to explain behavior under conditions of uncertainty arises from two considerations: (1) subjective feeling of imperfect knowledge when certain types of choices, typically involving commitments over time, are made; (2) the existence of certain observed phenomena, of which insurance is the most conspicuous example, which cannot be explained on the assumption that individuals act with subjective certainty.

Kenneth Arrow [R21.5, p. 44]

The world of human operations relates to three important time stages of past, present and future that are intimately connected for human progress. This is the *time trinity*. The history of this progress is the work of success-failure decision-choice actions which are governed by some rationality that constitutes a processing operator on the available knowledge input into the decision-choice modulus to obtain an output as an element of the evolving history. The time trinity presents us with uncertainty that houses contradictions and paradoxes as indispensable elements of human historic journey where information-knowledge structure constitutes the wind that drives the ship of human progress to sail in trouble waters with uncertain outcome. This information-knowledge structure is the connector of past, present and future as they relate to the behavior of cognitive agents. The past presents us with the problem of perceptive knowledge; the present provides us with the challenges of decision-choice actions and the future presents us with expectations of current decision-choice actions that are locked in the jacket of possibilistic belief system. The decision-choice outcomes are defined by probabilistic belief system and expectations of actualized outcomes on the basis of complex system of beliefs.

The possibilistic and probabilistic belief systems through decision-choice actions present potential and actual risks of varying intensity and scope to decision-choice agents. The uncertainty, expectation and risk have continuity with time where decision-choice agents must understand the past-present-future dynamics of potential-actual duality. This understanding must be abstracted from the construct of information-knowledge structure in support of possibilistic and probabilistic belief systems for cognitive substitution-transformation processes on the path of human progress in its complexity. The complex relationships among past, present and future and their impacts on the relationships among information, knowledge and rationality require of us critical reasoning and understanding. In this direction, the statement by Frank Knight is helpful in understanding the time trinity of past, present and future on one hand and the decision-choice process on the other. *"We do not perceive the present as it is and in its totality, nor do we infer the future from the present with any high degree of dependability, nor yet do we accurately know the consequences of our own actions"* [R4.34, p.202].

Generally, uncertainty, expectations and risk are derived from justified possibilistic and probabilistic belief systems. The justifications of these belief systems are derived from the information-knowledge structure that contains limitations of vagueness, ambiguities, incompleteness in the information and defective knowledge-acquisition capabilities. In understanding decision-choice actions, a number of questions tend to arise. What do we know from the past?

What is the relationship of what we know to the expected in the future? What effect does the expected have on the current decisions? What is uncertainty and is this uncertainty one or more than one type? What are the possible relationships among uncertainty, expectations and risk? How best can risk be defined and to what extent is the risk definition related to concepts of costs and benefits? Are there as many risks as there are uncertainty types? Are vagueness and ambiguities factors of uncertainty and hence generate risk? What are the generators of paradoxes in decision-choice theories under probabilistic reasoning in the classical paradigm?

The search for answers to these questions is one of the motivations to examine the epistemic foundations of uncertainty, expectations and risk under conditions of decision-choice rationality in knowledge production process as we move from the potential to the actual through the possible and the probable. This monograph is the result of this examination from the knowledge-production viewpoints of classical and fuzzy paradigms. The focus is on the nature of decision-choice rationality at the presence of uncertainty, expectations and risk; and how these three conceptual elements shape the use of conditions of information-knowledge structure in the general decision-choice process.

This monograph is the third in the sequence of a search for meaning and role of fuzzy reasoning and conditions of soft computing in differently defined environments in the different knowledge sectors. It may also be seen as a continuation of a conceptual system developed in my two volumes on cost-benefit analysis and theory of fuzzy decisions where the emphasis was on cost-benefit rationality [R7.35], [R7.37]. In those two volumes, the idea is to expand the conceptual system and algorithmic toolbox of cost-benefit analysis to all areas requiring, decision, choice and policy with the view that the essential tool for modern policy research and analysis is cost-benefit analysis. The objective in this volume, however, is to specify the foundations of fuzzy reasoning that will allow us to deal with conditions of vagueness and ambiguities in uncertainty and risk analysis through the methods and techniques of approximate reasoning and fuzzy rationality from the framework of fuzzy paradigm. The work is to examine applicable areas of probabilistic measures of risk and their uses in analytical reasoning in understanding decision-choice behavior in general human conditions. Similarly, it is intended to examine the applicable areas of fuzzy reasoning in decision-choice activities involving risk in substitution-transformation processes that account for time, quantity and quality in the movement from the potential through the possible, and the

probable to the actual. The essential epistemic point here is that probabilistic reasoning alone is not sufficient to characterize uncertainty, expectations and risk. Furthermore, probabilistic reasoning within the classical paradigm, offers us very little, if any, to accurately examine behavior under uncertainty and risk when vagueness and ambiguities characterize the variables under decision-choice action and logical operations.

My hope is that these three treaties on the epistemics on fuzzy rationality and paradigm will be, at least, a defining entry point into theory of knowledge that combines exact and inexact sciences by utilizing the toolbox of fuzzy paradigm. The vision is that some intellectually closed eyes will be opened, especially those of mathematical skeptics and believers of the supremacy of exact science and absolute truth. The added utility of these treaties is to make explicit, the difficult and relatively virgin areas that require extensive research on mathematical theories of fuzzy phenomena. Such areas include fuzzy-stochastic topological space with fuzzy-random variable and stochastic-fuzzy topological space with random-fuzzy variable. The fuzzy-random variable will allow us to deal with imprecise, or vague or ambiguous probabilities. The random-fuzzy variable will allow us to deal with random fuzzy behaviors. The knowledge of the logic and mathematical tools obtained from these topological spaces will provide new and powerful tools for areas like economics, medical sciences, psychology and others that we characterize as inexact sciences as well as some unresolved problems in exact sciences that are plagued with vagueness.

The subject area of fuzzy paradigm is relatively new in logic and mathematical reasoning. It is a paradigm shifting, as such; the monographs involve a number of new and controversial ideas. This paradigm shifting may be understood and appreciated from the duality viewpoint of exact and inexact sciences which is viewed here as mutually supporting in the knowledge-creation process. The introduction of some of the controversial ideas is also intentional. From my entry into Temple University and my course works in symbolic logic, mathematical analysis, philosophy of science and my encounter with economic theories and their application, the position of the Aristotelian logic and the principle of excluded middle were troubling. My background in African thought system instructs me that the elements in the universal system are relational in existence and every statement or proposition is relationally true or false; and that, truth and falsity exist in the same statement or hypothesis as opposing characteristics or attributes which must be reconciled by decision-choice action. An introductory exposition on this approach to theory of knowledge is provided in my monograph entitled *polyrhythmicity* [R20.21].

My encounter with multi-valued logic did not relieve me of this troubling logical form of verification and acceptance of truth. I sought some relief from statistical sciences, but my encounter with it and probabilistic reasoning, based on logic of classical paradigm and the principle of exactness, did not help my analytical concerns. It is these concerns that drove me to examine the structure of fuzzy logic, fuzzy sets, systems and fuzzy mathematics in terms of possible analytical relieve. The process of my encounter with the whole fuzzy research program and fuzzy paradigm has been described in my volumes on cost-benefit analysis and the theory of fuzzy decisions [R7.35], [R7.36] and my initial epistemic views on decision-choice rationality is provided in [R17.22].

The question that faces the community of scholars and scientists in the academic and research institutions is whether there is a unified logic and mathematics that will allow a consistent development towards unified knowledge systems that are both exact and inexact in reasoning. For reasons of consistency social acceptance of knowledge element should have the same criteria. The search for an answer to this question encounters series of difficulties with the framework of the classical paradigm. However, our ability to develop logic and mathematical techniques to advance the unification of inexact and exact sciences is the challenge of our time for speeding up the gains in systemicity, cybernetics, artificial intelligence, robotics, informatics, economics, social system's management, psychology, medical sciences and others. By advancing the epistemic structure of fuzzy rationality a case is made that the needed logic and mathematical techniques can be found in the framework of fuzzy paradigm when the epistemological foundations are understood.

Let us keep in mind that information is not knowledge and that uncertainty-risk reduction takes place on the basis of relevant knowledge. The fact of our current time is that information search and storage are such that we are constantly dealing with complex social system with information deficiency and overflow that make relevant knowledge more difficult to construct on the basis of the classical paradigm. The result of this increasing difficulty is to increase our problem in using information-knowledge structure for risk reduction and to increase decision-choice benefit. Here appears the importance of the statement: *We now have a new king of cost-benefit analysis, namely, benefit-risk analysis. The risk of a disutility is itself a cost and a proper subject for measurement along with the direct costs of the usual resource-using type. Similarly, a reduction in risk is to be counted as a benefit* [R21.37, p.20]. The information deficiency is not due to quantity limitation but quality deficiency

generated by vagueness, ambiguities, inexactness and other relevant qualitative elements that may be subjectively defined. The logic and techniques for processing such deficient information to obtain relevant knowledge to reduce social and natural risks are part of the need for understanding the epistemic foundations of fuzzy paradigm and corresponding rationality. Just as we need rationality postulate to measure probabilistic uncertainty and corresponding risk, we need a rationality postulate to deal with fuzzy uncertainty and associated risk. Both of them are not the same, and hence require different logical and mathematical tools for their analyses.

Given the task at hand, the monograph is organized in six chapters. Chapter One examines the nature of uncertainty and how it relates to information and knowledge in order to allow us to establish categorial uncertainties, zones of epistemic accessibilities and ignorance. The relationship between past-present-future structure on one hand and information-knowledge-decision structure on the other hand is presented and analyzed in relation to expectations formation towards decision-choice actins. The concept and logical technique of *knowledge square* are introduced in a manner that allows explication of possibility and probability to be advanced and related to the concepts and contents of possibility and probability indexes. The chapter is concluded with discussions on the relational structures among decision-choice rationality, uncertainty and expectations which are then related to the presence of vagueness, ambiguity, inexactness and the analytical structure of fuzzy optimal decision-choice rationality.

Chapter Two analyzes the nature and structure of classical sub-optimal decision-choice rationality relative to the classical optimal decision-choice rationality and the conditions that give rise to sub-optimal decision-choice behavior in the classical decision-choice system. The idea of equating classical sub-optimal decision-choice rationality to decision-choice irrationality is dismissed as logically indefensible. The judgment of whether there is decision-choice rationality is itself a decision-choice governed by some selected criterion conditions also defined by a rationality. The concept and content of classical sub-optimal rationality are given their epistemic structures and related to fuzzy optimal rationality. It is then argued that fuzzy optimal decision-choice rationality provides a covering over both classical optimal decision-choice rationality and sub-optimal rationality. The decision-choice theories that give rise to conditions of classical optimal and sub-optimal decision-choice rationalities are argued to be dealing with rationality as an ideal state of decision-choice processes conditional on the given optimal rationality as an attribute of decision-choice agents.

The rise of classical sub-optimal rationality in decision-choice process is explainable by the presence of vagueness, ambiguities, subjectivity and other qualitative factors in the information-knowledge structure and reasoning patterns of decision-choice agents. The process of epistemic explanation and resolution of the effects of vagueness, ambiguities, subjectivities and other qualitative factors that give rise to classical sub-optimal rationality leads to the examination of the roles that *fuzzification* and *defuzzification* play in general and specific cases using fuzzy logical reasoning in decision-choice processes. An example is given to show the differential natures in the use of classical and fuzzy mathematical reasoning in the computational space of decision-choice models. The chapter is concluded with discussions and examination of relational structure of fuzzy optimal decision-choice rationality, contradictions and benefit-cost rationality in decision-choice processes.

In chapter Three, we present and deal with the structure of critical and important problems of ambiguity, vagueness and other qualitative factors in decision-choice process and their impacts on the understanding of uncertainty and risk and their relationships to fuzzy optimal decision-choice rationality. The role of the concept of probability in understanding ambiguity in decision-choice process is analyzed in terms of strengths and weaknesses of classical and fuzzy paradigms. The consideration and analysis of simultaneous presence of fuzziness and randomness in the information-knowledge structure lead to the fuzzy-stochastic partition of the decision-choice space into four zones of *risk-free*, *stochastic risk*, *fuzzy risk* and combination of fuzzy and stochastic risk to obtain *fuzzy-stochastic risk* and *stochastic-fuzzy risk*.

The epistemic analysis of the differences, similarities and relationships among the risk cohorts are individually and collectively discussed where each cohort is relationally structured in terms of total uncertainty, cost-benefit balances and decision-choice motivation conditions. The epistemic structure of the organic paradigm of decision-choice theories is introduced, analyzed and synthesized for clarity of the relational structure of cost-benefit, motivation, knowledge, optimal rationality as an attribute of decision-choice agents and as an ideal state of the decision-choice process The chapter is concluded with reflective discussions on the relative nature of rationality in economics, psychology and decision-choice theories in general. It is then argued that economic theories on decision-choice behavior implicitly assume optimal rationality as an attribute of decision-choice agents and then investigate conditions of rationality as ideal state of decision-choice process while psychological

theories on decision-choice behavior relate to validity of rationality as an attribute of decision-choice agents.

Chapter Four is devoted to the analysis of epistemic problems of concepts and measurement of information, uncertainty and risk in decision-choice processes. The relative differences and similarities of classical theories under uncertainty and theories on risk are then discussed and related to our current structure of theories of information where the central focus of theories of risk and information are argued to be not only similar but find epistemic expressions in stochastic uncertainties, probability space and measures of probability. Epistemic definitions of uncertainty and risk are provided for explication that allows us to introduce concepts of pure uncertainty and risk, principle of information sufficiency, principle of knowledge sufficiency, stochastic uncertainty, fuzzy uncertainty, stochastic risk, and fuzzy risk with complications that they present for both analytical structures of classical and fuzzy decision-choice rationalities.

The relationships among these categories of uncertainty and risk are then related to the spaces of potential, possibility, probability and actual. It is then argued that complete risk is associated with the space of potential, fuzzy risk is associated with the possibility space, stochastic risk is associated with probability space and realized total risk is associated with the space of the actual and related to *knowledge-risk square*. The meaning of the concepts of necessity and accident, and the roles that they play in the substitution-transformation processes are discussed in relation to risk. Necessity and accident are then argued to be characteristics, first of possibility space and then of the probability space. Connections are established among risk, freedom and decision-choice rationality leading to a discussion on what decision-choice agents' freedom means under uncertainty and risk as they relate to categories of necessity and accidents. The chapter is concluded with epistemic analysis of the *principle of compatibility of necessity and freedom* in categorial dynamics of actual-potential duality in substitution-transformation processes, and how they help or hinder our understanding of decision-choice rationality under uncertainty and risk given cognitive agents.

In Chapter Five is critical reflections on some decision-choice theories on uncertainty and risk are examined where the theories are grouped into those concern with stochastic uncertainty, probabilistic belief and stochastic risk on one hand and those concerned with fuzzy uncertainty, possibilistic belief and fuzzy risk. The former belongs to the classical class of decision-choice theories with conditions of classical optimal decision-choice rationality while the latter belongs to the non-classical class of decision-choice theories of fuzzy

type with fuzzy optimal decision-choice rationality. The structure and form of the classical decision-choice theories on stochastic uncertainty and risk and the corresponding decision-choice rationality are analyzed and synthesized in relation to their contributions to our understanding of stochastic risk and behavior under stochastic risk. The concepts of event, outcome, uncertain event, risk, risky event and risky outcome receive fresh explication in terms of deficient information-knowledge structure as it relate to classical decision-choice rationality. A framework is then created to critically examine the strengths and weakness of the conditions of classical optimal decision-choice rationality under uncertainty and risk.

The framework involves the relational structure of logical principles of sufficient reason and insufficient reason and the role these principles play in the construct of decision-choice theories on uncertainty and risk as we traverse from the potential to the actual through the possibility and probability. These two principles are further supported by seven others such as principles of sufficient justification, cause, analytical sufficiency, excluded middle and others, as they affect the outcome of the use of methods and techniques of constructionism and reductionism. The conceptual and epistemic foundations of fuzzy risk and decision-choice rationality are presented and discussed by first partitioning the space of decision-choice variables into exact variable, stochastic variable, fuzzy variable and either fuzzy-stochastic or stochastic-fuzzy variable where two mathematical spaces of analytical interests are referenced. The fuzzy risk category is then partitioned into sub-categories of non-stochastic fuzzy risk, fuzzy-stochastic risk and stochastic-fuzzy risk and then related to the conditions of fuzzy optimal decision-choice rationality and approximate reasoning for soft computing.

The epistemic analysis presents total uncertainty and total risk as equal to the sum of stochastic uncertainty and fuzzy uncertainty, and the sum of fuzzy risk and stochastic risk respectively that must receive attention in our theories on risk and uncertainty. The phenomenon of risk is then conceptualized in terms of *natural risk* and *social risk* with explications that allow the differences and similarities to be established through the corresponding adaptation properties where risk is not characteristic of nature but conceptual element of cognitive agents The discussions and analytical works on the concepts, epistemic framework, and foundations of risk-engineering, social risk-engineering, natural risk-engineering with comparative analyses to risk-taking conclude the chapter. The essential point that must be noted is that total risk is the sum or a weighted sum of fuzzy and stochastic risks almost all important decision-

choice actions are undertaken under fuzzy-stochastic or stochastic-fuzzy risk. The mathematical knowledge on the behaviors of fuzzy-stochastic topological space with fuzzy-random variable, and stochastic-fuzzy topological space with random-fuzzy variable is not readily available to us to fully pursue the analysis of risk conditions and measures in these spaces.

Chapter Six concludes the analytical structure of the epistemic discussions of this monograph regarding problems and relational understanding of uncertainty, expectations and risk in decision-choice process. It is mostly devoted to examining the structure and form of paradoxes that have risen in classical decision-choice theories and models. All the paradoxes in classical decision-choice theories and the classical paradigm itself come to us as either temporary or permanent, it is argued. Permanent paradoxes are attributed to the presence of problems of subjective phenomena, vagueness, ambiguities and non-acceptance of logical duality in decision-choice behavior and knowledge construction and reduction. Discussions are made as to how the logic and analytical challenges of these paradoxes may be met with the toolbox of fuzzy paradigm and the methods and techniques of fuzzy optimal decision-choice rationality. The Arrow's paradox is subjected to an epistemic examination within the framework of the fuzzy paradigm for the understanding of the conditions that give rise to it as a permanent paradox in the classical paradigm as well as the decision-choice sub-space that the paradox was created.

A suggestion for its resolution is put forward. The relational structures of utility theory, probability and paradoxes in the classical decision-choice theories are analyzed. The role played by *deficient information-knowledge* structure in the rise of paradoxes is discussed through the analytical structure of fuzzy optimal decision-choice rationality and how the methods and tools of fuzzy optimal decision-choice rationality and the toolbox of fuzzy paradigm can assist us to resolve some, if not all, the paradoxes. The Ellsberg's paradox and Savage axioms are singled out for a critical examination where the contributions by the presences of the phenomena of subjectivity and vagueness and their implications on optimal decision-choice rationality are examined in relations to Frank Knight's measurable and unmeasurable uncertainties [R4.34].

The problem of Ellsberg is reframed to show its analytical defect relative to the decision-choice axioms of Savage by examining the problem in both possibility and probability spaces. It is then argued that the paradox arises by using two incompatible analytical frames. The Savage frame is in the probability space conditional on the assumption of existence of required possibility space. The choice variable here is the classical exact random variable. The Ellsberg frame is derived from both the possibility and probability spaces. The choice

variable is fuzzy-random variable requiring a non-classical framework for its analytics. The understanding and appreciation of the differences are very important in order to understand the Ellsberg's paradox. Savage deals with existence of probabilistic belief while Ellsberg deals with simultaneous existence of possibilistic and probabilistic beliefs. Reflections and criticisms of some suggested resolutions of the Ellsberg's paradox are provided with a conclusion that the use of correction and adjustment factors within the classical paradigm is logically unjustifiable. On the basis of the critique, a resolution of the Ellsberg's paradox is provided through the use of the toolbox of fuzzy paradigm and methods and techniques of fuzzy optimal decision-choice rationality. A numerical example is given to illustrate the nature of the resolution of the Ellsberg's paradox. The chapter is concluded with an epistemic examination of relational structure of fuzzy optimal decision-choice rationality and paradoxes in general and how the methods and techniques of the *knowledge square* and *quadrangle logical pyramids* will assist in our critical understanding of paradoxes as we seek knowledge of the process of moving from the *potential* to the *actual* through the *possible* and the *probable.*

In general and specific frameworks, the phenomena of uncertainty, expectations and risk must retain their inter-relational foci with themselves, information-knowledge structure and decision. In this respect, the analytical process on rationality should not lead to a situation where the explicate words are devoid of the concepts of the phenomena in such a way as to create phantom problems or irresolvable paradoxes. The reflection by Claude Bernard is useful at this point.

> *In creating a word to define a phenomenon, the idea it expresses is generally specified at that time together with its exact meaning. However, with the passage of time and the progress of science, the meaning of the word changes from but keeps its initial significance for others. As a result there is often such a discordance that persons employing the same word mean very different ideas. Our vocabulary is only approximate and so imprecise, even in science, that if we focus on words rather than phenomena, we stray quickly from reality. Science can only suffer when we discuss to keep a word which can only induce error because it does not convey the same meaning to all. Let us conclude that one must always focus on phenomena and view the word only as an expression void of meaning if the phenomena or if it happens that they do not exist.*

> *The mind of course moves systematically, which explains why we tend to reach an agreement on words rather than on phenomena. This leads experimental criticism in the wrong direction, confuses issues, and suggests the existence of dissidences; but these relate most often to the interpretation of phenomena, instead of the existence of facts and their true importance.*
>
> Claude Bernard [R21.16, p.5])

The statement by Bernard may be read in conjunction with that of Faraday in relation to cognitive anchorage on the road to discovery of scientific truth. We must always guide ourselves against the tyranny of established schools of thought and the acceptable boundaries of reasoning.

> *The scientist should be a man (person) willing to listen to every suggestion, but determined to judge for himself. He should not be biased by appearances; have no favorite hypothesis; be of no school; in doctrine have no master. He should not be a respecter of persons but of things. Truth should be his primary object.*
>
> Michael Faraday, [R21.16, p. 6]

Acknowledgements

The epistemic foundations of analysis of uncertainty, expectations, risk and the theory of knowledge are intimately connected to the theory of rational process of human action and rules of reason. Any serious analytical work on human understanding of nature and society and epistemic justification of belief system that correspond to rationality cannot neglect these phenomena. There are many works on these phenomena which we must be grateful. The phenomena of fuzziness and approximate reasoning as they relate to expectations, uncertainty and risk have received almost no attention. The works, however, by a number of scholars and researchers on fuzzy logic, mathematics, decision theory and category theory under fuzzy information-knowledge constraint provide us with a new toolbox for better understanding of rational behavior. We are thankful for all these dedicated people. However, the greatest danger to the progress of fuzzy research, we may note, is ideological and scientific credulity that finds expression in the classical paradigm with the principle of exactness that ties uncertainty, expectation and risk to only probabilistic belief, probability and stochastic processes. The danger may be diminished by acknowledging the fundamental idea that the vocabularies of our

languages and linguistic reasoning are approximations that require explications in communications. Extreme explication, however, may increase the irrelevance and misunderstanding of the communication process leading to increased penumbral region of decision-choice actions. In this respect, in the development of fuzzy paradigm, enhancement of its logic, expansion of its mathematical domain, and the integration of fuzziness and randomness, we are thankful for all researchers and scholars who have freed themselves from the ideological grip of the classical paradigm. This monograph has benefited from their works and contributions.

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