

# Numerical Methods in Multidimensional Radiative Transfer

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Editors

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 Springer

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ISBN 978-3-540-85368-8

e-ISBN 978-3-540-85369-5

DOI 10.1007/978-3-540-85369-5

Library of Congress Control Number: 2008937506

Mathematics Subject Classification (2000): 35M10

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*Cover design:* VTEX, Vilnius

Printed on acid-free paper

987654321

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## Preface

Traditionally, radiative transfer has been the domain of astrophysicists and climatologists. In nuclear technology one has been dealing with the analogous equations of neutron transport. In recent years, applications of radiative transfer in combustion machine design and in medicine became more and more important.

In all these disciplines one uses the radiative transfer equation to model the formation of the radiation field and its propagation. For slabs and spheres effective algorithms for the solution of the transfer equation have been available for quite some time. In addition, the analysis of the equation is quite well developed. Unfortunately, in many modern applications the approximation of a 1D geometry is no longer adequate and one has to consider the full 3D dependencies. This makes the modeling immensely more intricate. The main reasons for the difficulties result from the fact that not only the dimension of the geometric space has to be increased but one also has to employ two angle variables (instead of one) and very often one has to consider frequency coupling (due to motion or redistribution in spectral lines). In actual calculations this leads to extremely large matrices which, in addition, are usually badly conditioned and therefore require special care. Analytical solutions are not available except for very special cases.

Although radiative transfer problems are interesting also from a mathematical point of view, mathematicians have largely neglected the transfer equation for a long time. As a consequence, in each discipline various codes have been developed that—although they mostly served their purpose—were not optimal with respect to computing time and memory requirement as well as accuracy. Furthermore, the appropriate analytical basis had hardly been worked out.

It was therefore fortunate that the Deutsche Forschungsgemeinschaft for many years supported a graduate school and a special research project in Heidelberg in which mathematicians and astrophysicists collaborated on algorithms for the solution of the multidimensional radiative transfer equation. In this framework in 1994 and 2003 two interdisciplinary workshops took place

in Heidelberg. During the latter, the idea evolved to publish a book containing a collection of papers that illuminate multifaceted aspects of radiative transfer and in particular the progress in the numerical solution of the multidimensional radiative transfer equation. It resulted in the present volume.

The editors thank the Deutsche Forschungsgemeinschaft (SFB 359) for its longterm support for developing modern algorithms for the solution of the 3D radiative transfer equation.

Heidelberg, August 2008

G. Kanschä  
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