Optimization and Multiobjective Control of Time-Discrete Systems

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Optimization and Multiobjective Control of Time-Discrete Systems

Dynamic Networks and Multilayered Structures

Springer

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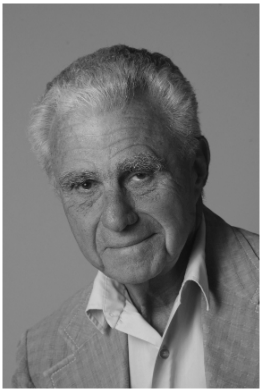
Foreword



Richard Bellmann developed a theory of dynamic programming which is for many reasons still in the center of great interest. The authors present a new approach in the field of the optimization and multi-objective control of time-discrete systems which is closely related to the work of Richard Bellmann. They develop their own concept and their extension to the optimization and multi-objective control of time-discrete systems as well as to dynamic networks and multilayered structures are very stimulating for further research.
Different perspectives of discrete control and optimal dynamic flow problems on networks are treated and characterized. Together with the algorithmic solutions a framework of multi-objective control problems is derived. The conclusion with a real world example underlines the necessity and importance of their theoretic framework. As they come back to the classical Bellmann concept of dynamic programming they stress and honor his basic concept without debase their own work.
Multilayered decision processes as part of the design and analysis of complex systems and networks will be essential in many ways and fields in the future.

George Leitmann, Berkeley June 2008

Preface

A relevant topic in modern control theory is concerned with multi-objective control problems and suitable extensions of methods for solving discrete problems that generalize classical ones. In this book an attempt is made to develop a mathematical framework for studying such classes of problems and to elaborate algorithms for solving them. The main concentration is addressed to multi-objective discrete control models with integral-time costs expressed by a trajectory when the starting and the final states of the dynamical system are fixed. Such models are formulated and studied by using game-theoretical concepts.

The dynamics of the system is assumed to be controlled by several actors (players) where each of them has the aim to optimize his own integral-time cost along the trajectory determined by vectors of control parameters chosen by all players together.

Pareto, Nash and Stackelberg optimization principles for the considered models are applied and new classes of dynamic cooperative, non-cooperative and hierarchical games, respectively, are defined.

The basic results are concerned with the determination of optimal stationary and non-stationary strategies of the players in the multi-objective control problems. Necessary and sufficient conditions for the existence of optimal strategies of the players are given and algorithms based on dynamic programming for finding such strategies are proposed.

Time-discrete systems with finite sets of states are studied. The dynamics of such systems are described by a directed graph in which each vertex corresponds to a dynamic state and the edges correspond to transitions of the system from one state to another. This fact allows us to formulate the considered control models on dynamic networks and to derive algorithms by using the so-called time-expanded network method. This method is developed for multi-objective control problems and dynamic optimal flow problems.

The book consists of five chapters.

In Chapter 1 we introduce multi-objective control problems with p players. The game-theoretical concept for classical discrete optimal control problems is studied and new classes of dynamic games are formulated. We introduce the multi-objective control problem for the non-cooperative as well as for the cooperative case. Stationary and non-stationary control parameters for these time discrete systems are determined. Theorems of the existence of Nash equilibria, Pareto optima and Stackelberg strategies in the considered dynamic games are proved. These are based on the concept of the so-called alternate players' control condition and the concept of dynamic games in positional form. Both were invented by Dmitrii Lozovanu as a main theoretical concept. The computational complexity is treated and the time expanded network is characterized. At the end hierarchical control problems are solved.

Chapter 2 is devoted to max-min discrete control problems and to the solution of zero-sum dynamic games on networks. Necessary and sufficient conditions for the existence of saddle points in such games are given and algorithms for determining the optimal strategies of the players are derived. The chapter begins with discrete control problems and finite antagonistic games. Max-min control problems with infinite time horizon and zero-sum games on networks are introduced. In the main part results for an arbitrary network are derived. The most important results of this chapter are concerned with finding the optimal stationary strategies in cyclic games. Algorithms based on dynamic programming and the dichotomy method for finding optimal strategies are proposed.

In Chapter 3 we extend and generalize the models developed in the first part of the book. We introduce discrete control problems with varying time of states' transitions. An algorithm for solving a single objective control problem is presented. Discrete control problems with certain cost functions of system passages that depend on the transition time of states' transitions are introduced. Furthermore, the control problem with transition time functions on the edges is solved.

In the main part of this chapter multi-objective control problems of time discrete systems with varying time of states' transition are considered. We present an algorithm for solving the discrete optimal control problem with infinite time horizon as well as with varying time of states' transitions. The chapter concludes with a general approach for algorithmic solutions of discrete optimal control problems. Within these game-theoretic extension the non-cooperative and the cooperative case is treated. At the end the new special concept of Pareto/Nash equilibria for multi-objective control and of a Pareto-Stackelberg solution are characterized and interpreted.

Chapter 4 is devoted to optimal dynamic flow problems which generalize discrete control problems on networks. The time-expanded network method for solving optimal dynamic multi-commodity flow problems is developed. The chapter begins with single commodity dynamic flow problems and the presentation of the time expanded network method for their solving.

We consider a dynamic model with flow storage at nodes and introduce integral constant demand supply functions. We extend this approach by minimum cost flows and develop a suitable algorithm for solving the main dynamic flow problem. In the main part multi-commodity dynamic flow problems and algorithms for their solving are treated. The chapter ends with a few generalizations, especially with an algorithm for solving the maximum dynamic multi-commodity flow problem. Finally, a game-theoretic approach for dynamic flow problems is treated.

Chapter 5 refers to applications and related topics. In the first chapters the theoretical framework was treated in detail. The work of Richard Bellmann is extended and several algorithms and game-theoretic concepts are developed. The authors use this framework to model a general multilayered decision process on networks (the abbreviation is *MILAN*). As a an example the so-called Technology Emission Means (TEM-) Model which was developed by Stefan Pickl is extended and embedded into a general multilayered decision process.

In the first part of this chapter, the TEM model is introduced as a time-discrete system which should be used for resource planning processes. The problem of fixed point controllability and null-controllability is introduced as well as the determination of the optimal investment parameter. A game-theoretical extension and another interpretation of the Bellmann functional equation leads directly to an applied multilayered decision process and terminates the book.

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