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# Adaptive Backstepping Control of Uncertain Systems

Nonsmooth Nonlinearities, Interactions  
or Time-Variations

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*To Feng Zhou, Lingfang Ma, Xiaozhong Shen and Zhile Shen*

J. Zhou

*To Xiu Zhou, Wen, Wendy, Qingyun and Qinghao*

C. Wen

# Preface

This book presents new methodologies for the design and analysis of adaptive control systems based on the backstepping approach. Our emphasis is on dynamic uncertain systems with nonsmooth nonlinearities, such as backlash, dead-zone, hysteresis and saturation, or time-varying parameters, or interactions.

The backstepping approach, a recursive Lyapunov-based scheme, was proposed in the beginning of 1990s. With this method the construction of feedback control laws and Lyapunov functions is systematic, following a step-by-step algorithm. Backstepping can be used to relax the matching condition, which blocked the traditional Lyapunov-based design. A major advantage of backstepping is that it has the flexibility to avoid cancellations of useful nonlinearities and achieve regulation and tracking properties. The technique was comprehensively addressed by Krstic, Kanellakopoulos and Kokotovic in [1]. However, there is still no monograph available to address problems such as the handling of nonsmooth nonlinearities, time varying parameters and system interactions using this approach.

Nonsmooth nonlinearities such as dead-zone, backlash, hysteresis and saturation are common in industrial control systems, such as mechanical, hydraulic, biomedical, piezoelectric, and physical systems. Such nonlinearities are usually poorly known and may vary with time, and they often limit system performance. An effective control method should be able to accommodate such common practical nonsmooth nonlinearities. In practice, system parameters are changing with time. Parameter time-variations may arise from linear approximations along different motions and may be due to unmodelled dynamics (for instance friction parameters, electric resistances, inertias) and also some other factors such as changes in environmental conditions. For such systems, the control problem is very complicated and becomes even more difficult to deal with when time-varying parameters are unknown. In the control of large scale systems, one usually faces poor knowledge on plant parameters and interactions between subsystems. Decentralized adaptive control strategy is an efficient and effective way for controlling these systems with large amount of uncertainties. Decentralized adaptive

controllers are designed independently for local subsystems by using local available signals for feedback. The major challenge is how to compensate the effects of ignored interactions among subsystems.

In this book, we will present research results on adaptive control of such systems with the backstepping based technique, including theoretical success and practical development such as the approaches for stability analysis and the improvement of system tracking and transient performance. These results are given in two parts:

- The first part involves designing and analyzing adaptive backstepping controllers for multi-input multi-output (MIMO) systems, time-varying systems, or larger scale systems. Newly developed strategies are presented. The designed controllers are shown to guarantee all signals bounded in the system and yield good transient and tracking performance.
- In the second part, we will consider systems with four types of nonsmooth nonlinear characteristics, namely backlash, dead-zone, hysteresis and saturation. It will be shown how these four nonsmooth nonlinear characteristics can be adaptively compensated and how desired system performance is achieved, by incorporating the backstepping technique with other methodologies such as the inverse technique. The proposed adaptive control schemes are shown to ensure the stability of the resulting control system. With these schemes, system performances can be precisely characterized as functions of design parameters and thus is tunable in certain sense by designers. Each of these nonsmooth characteristics is considered individually and systematically. The developed adaptive control methodologies are also applied to the control of base isolation mechanism and piezo-positioning mechanism.

This book is helpful to learn and understand the fundamental backstepping schemes for state feedback control and output feedback control. It can be used as a reference book or a textbook on adaptive control with applications for students with some background in feedback control systems. The book is also intended to introduce researchers and practitioners to the area of adaptive control systems involving the treatment on nonsmooth nonlinearities, interactions and time varying parameters. Researchers, graduate students and engineers in the fields of electrical engineering, control, applied mathematics, computer science and others will benefit from this book.

We are grateful to Nanyang Technological University and Norwegian University of Science and Technology for providing plenty of resources for our research work. Jing Zhou would like to acknowledge StatoilHydro for their support.

We would also like to express our deep sense of gratitude to our parents and families who have made us capable enough to write this book. Jing Zhou is very grateful to her parents, Feng Zhou and Lingfang Ma, and her husband, Xiaozhong Shen, for their care, understanding and encouragement. Changyun Wen is greatly indebted to his wife Xiu Zhou and his children Wen Wen, Wendy Wen, Qingyun Wen and Qinghao Wen for their constant support throughout these years.

Finally, we would like to thank the entire team of Springer publications for their cooperation and encouragement in bringing out the work in the form of monograph in such a short span of time.

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