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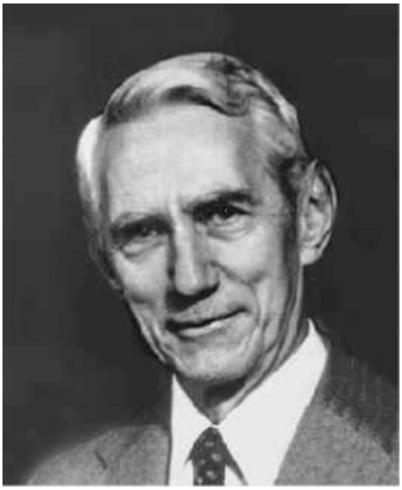
John von Neumann



Claude Shannon

Erwin Schrödinger

Dénes Petz

Quantum Information Theory and Quantum Statistics

With 10 Figures

Springer

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Preface

Quantum mechanics was one of the very important new theories of the 20th century. John von Neumann worked in Göttingen in the 1920s when Werner Heisenberg gave the first lectures on the subject. Quantum mechanics motivated the creation of new areas in mathematics; the theory of linear operators on Hilbert spaces was certainly such an area. John von Neumann made an effort toward the mathematical foundation, and his book "*The mathematical foundation of quantum mechanics*" is still rather interesting to study. The book is a precise and self-contained description of the theory, some notations have been changed in the mean time in the literature.

Although quantum mechanics is mathematically a perfect theory, it is full of interesting methods and techniques; the interpretation is problematic for many people. An example of the strange attitudes is the following: "*Quantum mechanics is not a theory about reality, it is a prescription for making the best possible prediction about the future if we have certain information about the past*" (G. 't' Hooft, 1988). The interpretations of quantum theory are not considered in this book. The background of the problems might be the probabilistic feature of the theory. On one hand, the result of a measurement is random with a well-defined distribution; on the other hand, the random quantities do not have joint distribution in many cases. The latter feature justifies the so-called quantum probability theory.

Abstract information theory was proposed by electric engineer Claude Shannon in the 1940s. It became clear that coding is very important to make the information transfer efficient. Although quantum mechanics was already established, the information considered was classical; roughly speaking, this means the transfer of 0–1 sequences. Quantum information theory was born much later in the 1990s. In 1993 C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W. Wootters published the paper *Teleporting an unknown quantum state via dual classical and EPR channels*, which describes a state teleportation protocol. The protocol is not complicated; it is somewhat surprising that it was not discovered much earlier. The reason can be that the interest in quantum computation motivated the study of the transmission of quantum states. Many things in quantum information theory is related to quantum computation and to its algorithms. Measurements on a quantum system provide classical information, and due to the randomness classical statistics can be used to estimate the true state. In some examples, quantum information can appear, the state of a subsystem can be so.

The material of this book was lectured at the Budapest University of Technology and Economics and at the Central European University mostly for physics and mathematics majors, and for newcomers in the area. The book addresses graduate students in mathematics, physics, theoretical and mathematical physicists with some interest in the rigorous approach. The book does not cover several important results in quantum information theory and quantum statistics. The emphasis is put on the real introductory explanation for certain important concepts. Numerous examples and exercises are also used to achieve this goal. The presentation is mathematically completely rigorous but friendly whenever it is possible. Since the subject is based on non-trivial applications of matrices, the appendix summarizes the relevant part of linear analysis. Standard undergraduate courses of quantum mechanics, probability theory, linear algebra and functional analysis are assumed. Although the emphasis is on quantum information theory, many things from classical information theory are explained as well. Some knowledge about classical information theory is convenient, but not necessary.

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Dénes Petz

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