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Parameter Estimation in Stochastic Differential Equations

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To the memory of my late grand father
and to my parents and brothers for their love and affection

Preface

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Charlotte, NC
January 30, 2007

Jaya P.N. Bishwal

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Basic Notations

| | |
|--------------------------------|---|
| (Ω, \mathcal{F}, P) | probability space |
| \mathbb{R} | real line |
| \mathbb{C} | complex plane |
| I_A | indicator function of a set A |
| $\xrightarrow{\mathcal{D}[P]}$ | convergence in distribution under the measure P |
| \xrightarrow{P} | convergence in probability P |
| a.s. [P] | almost surely under the measure P |
| P-a.s. | almost surely under the measure P |
| $a_n = o(b_n)$ | $\frac{a_n}{b_n} \rightarrow 0$ |
| $a_n = O(b_n)$ | $\frac{a_n}{b_n}$ is bounded |
| $X_n = o_P(b_n)$ | $\frac{X_n}{b_n} \xrightarrow{P} 0$ |
| $X_n = O_P(b_n)$ | $\frac{X_n}{b_n}$ is stochastically bounded, i.e., $\lim_{A \rightarrow \infty} \sup_n P\{ \frac{X_n}{b_n} > A\} = 0$ |
| □ | end of a proof |
| $A := B$ | A is defined by B |
| $A =: B$ | B is defined by A |
| \equiv | identically equal |
| \ll | absolute continuity of two measures |
| i.i.d. | independent and identically distributed |
| $\mathcal{N}(a, b)$ | normal distribution with mean a and variance b |
| $\Phi(\cdot)$ | standard normal distribution function |
| $X \sim F$ | X has the distribution F |
| w.r.t. | with respect to |
| r.h.s. | right hand side |
| l.h.s. | left hand side |
| $a \vee b$ | maximum of a and b |
| $a \wedge b$ | minimum of a and b |