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# **Control of Singular Systems with Random Abrupt Changes**

 Springer

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Dedicated to my wife Saida and my kids Imane, Ibtissama and Anas

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## Preface

Singular systems also referred to as descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems (see [43, 76]) represents an interesting class of dynamical systems since it combines differential equations and algebraic equations and generalizes the linear time invariant model which is extensively used in linear control theory. This class of systems has been used to model varieties of systems like economics, chemical processes, mechanics, electrical systems, etc. Singular systems have attracted a lot of researchers from the mathematical and control communities and a great number of fundamental notions and results in control and systems theory based on linear time-invariant systems with state space representation have been successfully extended to the class of singular systems. For more details on what it has been done on this subject we refer the reader to [3, 4, 51, 75, 122, 123, 127, 50, 109, 112, 122, 124, 126], and the references therein. More specifically, we tackled the stability and the stabilizability of the class of linear continuous-time singular systems and many results have been reported in the literature in the LMI setting. Among these results we quote those of [14]. The robust stability and the robust stabilization have also been studied. Many types of uncertainties have been considered among them we quote the norm bounded form, the linear fractional transformation (LFT, which generalizes the norm bounded type) and polytopic form.

The  $\mathcal{H}_\infty$  control and the filtering problems have also been treated and interesting results were reported in the literature. For the  $\mathcal{H}_\infty$  control, under the assumptions that the external disturbances have finite power or finite energy, a control law is designed to guarantee that the closed-loop system is regular, impulse-free and stable and at the same time assures the disturbance rejection with a given level  $\gamma > 0$ . For the filtering problem, the objective is to design a dynamical estimator that estimates the state vector that can be used in the control to make the closed-loop of the singular system regular, impulse-free and stable.

Some of the practical systems are stochastic or more specifically we can model them by a class of stochastic systems driven by continuous-time Markov chains that we will refer to as the class of stochastic systems with abrupt changes. This class of systems is more appropriate to model many practical systems, where random failures

and repairs and sudden environment changes may occur. For more detail on what it has been done on the subject, we refer the reader to [27], [96] and the references therein. This class of systems has also attracted a lot of researchers from both mathematical and control community. Many results on stochastic stability and stochastic stabilization have been reported in the literature. For more details on these results we refer the reader to [29, 26, 49, 60, 94] and the references therein, where different approaches have been used. The  $\mathcal{H}_\infty$  control problem was investigated in [45, 105], where sufficient conditions for the solvability of this problem was proposed. When time delays appear in a Markovian jump system, the results on stability analysis and  $\mathcal{H}_\infty$  control were also reported in [31], [24] and [25] for different types of time delays. For more detail on Markovian jumping systems with time delay, we refer the reader to [14, 27] and the references therein.

Our goal in this volume is to combine the class of singular systems with the one of systems with abrupt changes which will give the class of systems that we will refer to as the class of stochastic singular systems with abrupt changes. Our main objective in this volume is to treat the stability and the stabilization problems using different techniques. We will also handle the filtering problem.

The rest of this book is organized as follows. In Chap. 1, the different problems are stated and the necessary assumptions are given. Chapter 2 deals with the stability problem of the class of singular systems with random abrupt changes and LMI conditions are developed to check if a given system of this class of systems is piecewise regular, impulse-free and stochastically stable. The robust stability problem is also considered. Chapter 3 treats the stabilization problem and its robustness. State space controllers are considered and LMI design approaches are developed. Chapter 4 deals with the  $\mathcal{H}_\infty$  control for the class of singular piecewise deterministic systems. In Chap. 5, the static output stabilization is tackled and LMI results are developed for the class of singular systems with random abrupt changes. The robust case is also considered. Chapter 6 deals with observer-based output stabilization for the class of Markovian jump singular systems. In Chapt. 7 the filtering problem is considered and design procedures are proposed in the LMI formalism to solve the the  $\mathcal{H}_\infty$  filtering problem. In Chap. 8, the guaranteed cost control problem is tackled and LMI results are developed to synthesize the state feedback controller that makes the closed-loop piecewise regular, impulse-free and stochastically stable and at the same time guaranteed an upper bound for the cost for all admissible uncertainties. In Chap. 9, the mixed  $\mathcal{H}_2/\mathcal{H}_\infty$  control is tackled and design procedure is developed to synthesize a state feedback controller. Finally, Chap. 10 provides some tools that can be used to solve all the LMI conditions. It gives an idea to the reader on how to write his program under Matlab to solve the considered problem of this class of systems.

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