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The 1-2-3 of Modular Forms

Lectures at a Summer School in Nordfjordeid, Norway

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Preface

This book grew out of lectures given at the summer school on “Modular Forms and their Applications” at the Sophus Lie Conference center in Nordfjordeid in June 2004. This center, set beautifully in the fjords of the west coast of Norway, has been the site of annual summer schools in algebra and algebraic geometry since 1996. The schools are a joint effort between the universities in Bergen, Oslo, Tromsø and Trondheim. They are primarily aimed at graduate students in Norway, but also attract a large number of students from other parts of the world. The theme varies among central topics in contemporary mathematics, but the format is the same: three leading experts give independent but connected series of lectures, and give exercises that the students work on in evening sessions.

In 2004 the organizing committee consisted of Stein Arild Strømme (Bergen), Geir Ellingsrud and Kristian Ranestad (Oslo) and Alexei Rudakov (Trondheim). We wanted to have a summer school that introduced the students both to the beauty of modular forms and to their varied applications in other areas of mathematics, and were very fortunate to have Don Zagier, Jan Bruinier and Gerard van der Geer give the lectures.

The lectures were organized in three series that are reflected in the title of this book both by their numbering and their content. The first series treats the classical one-variable theory and some of its many applications in number theory, algebraic geometry and mathematical physics.

The second series, which has a more geometric flavor, gives an introduction to the theory of Hilbert modular forms in two variables and to Hilbert modular surfaces. In particular, it discusses Borcherds products and some geometric and arithmetic applications.

The third gives an introduction to Siegel modular forms, both scalar- and vector-valued, especially Siegel modular forms of degree 2, which are functions of three complex variables. It presents a beautiful application of the theory of curves over finite fields to Siegel modular forms by providing evidence for a conjecture of Harder on congruences between elliptic and Siegel modular forms.

Günter Harder came forward with this conjecture in a colloquium lecture in Bonn in 2003. He kindly allowed us to include his notes for this colloquium talk in Bonn on the subject. Even though the three lecture series are strongly connected, each of them is self contained and can be read independently of the others.

There is quote ascribed (perhaps apocryphally) to Martin Eichler, saying that there are five fundamental operations in mathematics: addition, subtraction, multiplication, division and modular forms. We hope this book will help convince newcomers and oldtimers alike that this is only partially an exaggeration.

Oslo, July 2007 *Kristian Ranestad*

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