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Alexander Balanov, Natalia Janson,
Dmitry Postnov, Olga Sosnovtseva

Synchronization

From Simple to Complex

With 150 Figures

 Springer

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To our parents

Preface

This book is written by scientists who live in different countries (United Kingdom, Denmark, Russia), but who have graduated from, and were established as researchers at the same place: The Laboratory of Nonlinear Dynamics, Department of Physics, Saratov State University, Russia. Being apart for many years, we have united in one team again to write this book. Why?

We aim to summarize both classical results that are crucial for the understanding of the concept of synchronization, and an up-to-date account of the accompanying fascinating phenomena. The main theme that runs throughout the book is that interaction between complex systems is governed by the same universal principles. We strive to explain the material in a way that the newcomers to the field would hopefully appreciate, namely,

- From simple calculations to advanced theoretical approaches
- From simple dynamics to complex behavior
- From mathematical and physical to general perspectives

Assuming only the basic knowledge of mathematics, our book takes the reader to the frontiers of what is currently known about this research area.

The classical approach to synchronization we have learned by heart during our regular and inevitably hot discussions, and most of the results on the new synchronization phenomena we obtained together. It is therefore difficult to separate scientific contribution and to compare the efforts made by each co-author, so we decided to arrange the list of authors in alphabetic order to emphasize an equal investment of their time, ideas and enthusiasm.

This book would not have been possible without the help of many people. First of all, we are deeply indebted to our teacher Prof. V.S. Anishchenko who has introduced us to Nonlinear World and who patiently taught us to properly speak the language of science. We are grateful to our teachers and colleagues Prof. V.V. Astakhov and T.E. Vadivasova for their active support and many invaluable discussions during the years. We extend our thanks to Prof. E. Mosekilde, Prof. P. McClintock, Prof. S.K. Han, who are our closest collaborators in the field of synchronization, and to Prof. N.-H. Holstein-Rathlou, Prof. D. Marsh, and Prof. H. Braun, with whom we have been enjoying collaborations in the field of modeling of biological systems. Our special thanks are due to Prof. E. Schöll who has encouraged us to write this book. We acknowledge fruitful discussions with our colleagues A. Pikovsky, M. Rosenblum, M. Zaks, J. Kurths, L. Schimansky-Geier, A. Neiman, A. Nikitin,

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