

Quantum Mechanics

Franz Schwabl

Quantum Mechanics

Fourth Edition

With 123 Figures, 16 Tables,

Numerous Worked Examples and 127 Problems

 Springer

Professor Dr. Franz Schwabl
Physik-Department
Technische Universität München
James-Franck-Strasse 2
85748 Garching, Germany
E-mail: schwabl@ph.tum.de

The first edition, 1992, was translated by Dr. Ronald Kates

Title of the original German edition: *Quantenmechanik* 7th edition
(Springer-Lehrbuch) ISBN 978-3-540-73674-5
© Springer-Verlag Berlin Heidelberg 2007

Library of Congress Control Number: 2007934044

ISBN 978-3-540-71932-8 4th ed. Springer Berlin Heidelberg New York
ISBN 078-3-540-43109-1 3rd ed. Springer Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable for prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media

springer.com

© Springer-Verlag Berlin Heidelberg 1992, 1995, 2002, 2007

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting and production: LE- \TeX Jelonek, Schmidt & Vöckler GbR, Leipzig
Cover design: eStudio Calamar S.L., F. Steinen-Broo, Pau/Girona, Spain

SPIN: 11766414 56/3180/YL 5 4 3 2 1 0 Printed on acid-free paper

Preface to the Fourth Edition

In this latest edition new material has been added, which includes many additional clarifying remarks to some of the more advanced chapters. The design of many figures has been reworked to enhance the didactic appeal of the book. However, in the course of these changes, I have attempted to keep intact the underlying compact nature of the book.

I am grateful to many colleagues for their help with this substantial revision. Special thanks go to Uwe Täuber and Roger Hilton for discussions, comments and many constructive suggestions on this new edition. Some of the figures which were of a purely qualitative nature have been improved by Robert Seyrkammer in now being computer-generated. I am very obliged to Andrej Vilfan for redoing and checking the computation of some of the scientifically more demanding figures. I am also very grateful to Ms Ulrike Ollinger who undertook the graphical design of the diagrams. It is my pleasure to thank Dr. Thorsten Schneider and Mrs Jacqueline Lenz of Springer for their excellent co-operation, as well as the L^AT_EX setting team for their careful incorporation of the amendments for this new edition. Finally, I should like to thank all colleagues and students who, over the years, have made suggestions to improve the usefulness of this book.

Munich, August 2007

F. Schwabl

Preface to the First Edition

This is a textbook on quantum mechanics. In an introductory chapter, the basic postulates are established, beginning with the historical development, by the analysis of an interference experiment. From then on the organization is purely deductive. In addition to the basic ideas and numerous applications, new aspects of quantum mechanics and their experimental tests are presented. In the text, emphasis is placed on a concise, yet self-contained, presentation. The comprehensibility is guaranteed by giving all mathematical steps and by carrying out the intermediate calculations completely and thoroughly.

The book treats nonrelativistic quantum mechanics without second quantization, except for an elementary treatment of the quantization of the radiation field in the context of optical transitions. Aside from the essential core of quantum mechanics, within which scattering theory, time-dependent phenomena, and the density matrix are thoroughly discussed, the book presents the theory of measurement and the Bell inequality. The penultimate chapter is devoted to supersymmetric quantum mechanics, a topic which to date has only been accessible in the research literature.

For didactic reasons, we begin with wave mechanics; from Chap. 8 on we introduce the Dirac notation. Intermediate calculations and remarks not essential for comprehension are presented in small print. Only in the somewhat more advanced sections are references given, which even there, are not intended to be complete, but rather to stimulate further reading. Problems at the end of the chapters are intended to consolidate the student's knowledge.

The book is recommended to students of physics and related areas with some knowledge of mechanics and classical electrodynamics, and we hope it will augment teaching material already available.

This book came about as the result of lectures on quantum mechanics given by the author since 1973 at the University of Linz and the Technical University of Munich. Some parts of the original rough draft, figures, and tables were completed with the help of R. Alkofer, E. Frey and H.-T. Janka. Careful reading of the proofs by Chr. Baumgärtel, R. Eckl, N. Knoblauch, J. Krumrey and W. Rossmann-Block ensured the factual accuracy of the translation. W. Gasser read the entire manuscript and made useful suggestions about many of the chapters of the book. Here, I would like to express my sincere gratitude to them, and to all my other colleagues who gave important assistance in producing this book, as well as to the publisher.

Munich, June 1991

F. Schwabl

Table of Contents

1. Historical and Experimental Foundations	1
1.1 Introduction and Overview	1
1.2 Historically Fundamental Experiments and Insights	3
1.2.1 Particle Properties of Electromagnetic Waves	3
1.2.2 Wave Properties of Particles, Diffraction of Matter Waves	7
1.2.3 Discrete States	8
2. The Wave Function and the Schrödinger Equation	13
2.1 The Wave Function and Its Probability Interpretation	13
2.2 The Schrödinger Equation for Free Particles	15
2.3 Superposition of Plane Waves	16
2.4 The Probability Distribution for a Measurement of Momentum	19
2.4.1 Illustration of the Uncertainty Principle	21
2.4.2 Momentum in Coordinate Space	22
2.4.3 Operators and the Scalar Product	23
2.5 The Correspondence Principle and the Schrödinger Equation	26
2.5.1 The Correspondence Principle	26
2.5.2 The Postulates of Quantum Theory	27
2.5.3 Many-Particle Systems	28
2.6 The Ehrenfest Theorem	28
2.7 The Continuity Equation for the Probability Density	31
2.8 Stationary Solutions of the Schrödinger Equation, Eigenvalue Equations	32
2.8.1 Stationary States	32
2.8.2 Eigenvalue Equations	33
2.8.3 Expansion in Stationary States	35
2.9 The Physical Significance of the Eigenvalues of an Operator	36
2.9.1 Some Concepts from Probability Theory	36
2.9.2 Application to Operators with Discrete Eigenvalues	37
2.9.3 Application to Operators with a Continuous Spectrum	38
2.9.4 Axioms of Quantum Theory	40

2.10	Additional Points	41
2.10.1	The General Wave Packet	41
2.10.2	Remark on the Normalizability of the Continuum States	43
	Problems	44
3.	One-Dimensional Problems	47
3.1	The Harmonic Oscillator	47
3.1.1	The Algebraic Method	48
3.1.2	The Hermite Polynomials	52
3.1.3	The Zero-Point Energy	54
3.1.4	Coherent States	56
3.2	Potential Steps	58
3.2.1	Continuity of $\psi(x)$ and $\psi'(x)$ for a Piecewise Continuous Potential	58
3.2.2	The Potential Step	59
3.3	The Tunneling Effect, the Potential Barrier	64
3.3.1	The Potential Barrier	64
3.3.2	The Continuous Potential Barrier	67
3.3.3	Example of Application: α -decay	68
3.4	The Potential Well	71
3.4.1	Even Symmetry	72
3.4.2	Odd Symmetry	73
3.5	Symmetry Properties	76
3.5.1	Parity	76
3.5.2	Conjugation	77
3.6	General Discussion of the One-Dimensional Schrödinger Equation	77
3.7	The Potential Well, Resonances	81
3.7.1	Analytic Properties of the Transmission Coefficient	83
3.7.2	The Motion of a Wave Packet Near a Resonance ...	87
	Problems	92
4.	The Uncertainty Relation	97
4.1	The Heisenberg Uncertainty Relation	97
4.1.1	The Schwarz Inequality	97
4.1.2	The General Uncertainty Relation	97
4.2	Energy–Time Uncertainty	99
4.2.1	Passage Time and Energy Uncertainty	100
4.2.2	Duration of an Energy Measurement and Energy Uncertainty	100
4.2.3	Lifetime and Energy Uncertainty	101
4.3	Common Eigenfunctions of Commuting Operators	102
	Problems	106

5. Angular Momentum	107
5.1 Commutation Relations, Rotations	107
5.2 Eigenvalues of Angular Momentum Operators	110
5.3 Orbital Angular Momentum in Polar Coordinates	112
Problems	118
6. The Central Potential I	119
6.1 Spherical Coordinates	119
6.2 Bound States in Three Dimensions	122
6.3 The Coulomb Potential	124
6.4 The Two-Body Problem	138
Problems	140
7. Motion in an Electromagnetic Field	143
7.1 The Hamiltonian	143
7.2 Constant Magnetic Field \mathbf{B}	144
7.3 The Normal Zeeman Effect	145
7.4 Canonical and Kinetic Momentum, Gauge Transformation ..	147
7.4.1 Canonical and Kinetic Momentum	147
7.4.2 Change of the Wave Function Under a Gauge Transformation	148
7.5 The Aharonov–Bohm Effect	149
7.5.1 The Wave Function in a Region Free of Magnetic Fields	149
7.5.2 The Aharonov–Bohm Interference Experiment	150
7.6 Flux Quantization in Superconductors	153
7.7 Free Electrons in a Magnetic Field	154
Problems	155
8. Operators, Matrices, State Vectors	159
8.1 Matrices, Vectors, and Unitary Transformations	159
8.2 State Vectors and Dirac Notation	164
8.3 The Axioms of Quantum Mechanics	169
8.3.1 Coordinate Representation	170
8.3.2 Momentum Representation	171
8.3.3 Representation in Terms of a Discrete Basis System	172
8.4 Multidimensional Systems and Many-Particle Systems	172
8.5 The Schrödinger, Heisenberg and Interaction Representations	173
8.5.1 The Schrödinger Representation	173
8.5.2 The Heisenberg Representation	174
8.5.3 The Interaction Picture (or Dirac Representation) .	176
8.6 The Motion of a Free Electron in a Magnetic Field	177
Problems	181

9. Spin	183
9.1 The Experimental Discovery of the Internal Angular Momentum	183
9.1.1 The “Normal” Zeeman Effect	183
9.1.2 The Stern–Gerlach Experiment	183
9.2 Mathematical Formulation for Spin-1/2	185
9.3 Properties of the Pauli Matrices	186
9.4 States, Spinors	187
9.5 Magnetic Moment	188
9.6 Spatial Degrees of Freedom and Spin	189
Problems	191
10. Addition of Angular Momenta	193
10.1 Posing the Problem	193
10.2 Addition of Spin-1/2 Operators	194
10.3 Orbital Angular Momentum and Spin 1/2	196
10.4 The General Case	198
Problems	201
11. Approximation Methods for Stationary States	203
11.1 Time Independent Perturbation Theory (Rayleigh–Schrödinger)	203
11.1.1 Nondegenerate Perturbation Theory	204
11.1.2 Perturbation Theory for Degenerate States	206
11.2 The Variational Principle	207
11.3 The WKB (Wentzel–Kramers–Brillouin) Method	208
11.4 Brillouin–Wigner Perturbation Theory	211
Problems	212
12. Relativistic Corrections	215
12.1 Relativistic Kinetic Energy	215
12.2 Spin–Orbit Coupling	217
12.3 The Darwin Term	219
12.4 Further Corrections	222
12.4.1 The Lamb Shift	222
12.4.2 Hyperfine Structure	222
Problems	225
13. Several-Electron Atoms	227
13.1 Identical Particles	227
13.1.1 Bosons and Fermions	227
13.1.2 Noninteracting Particles	230
13.2 Helium	233
13.2.1 Without the Electron–Electron Interaction	233

13.2.2	Energy Shift Due to the Repulsive Electron–Electron Interaction	235
13.2.3	The Variational Method	240
13.3	The Hartree and Hartree–Fock Approximations (Self-consistent Fields)	241
13.3.1	The Hartree Approximation	242
13.3.2	The Hartree–Fock Approximation	244
13.4	The Thomas–Fermi Method	247
13.5	Atomic Structure and Hund’s Rules	252
	Problems	258
14.	The Zeeman Effect and the Stark Effect	259
14.1	The Hydrogen Atom in a Magnetic Field	259
14.1.1	Weak Field	260
14.1.2	Strong Field, the Paschen–Back Effect	260
14.1.3	The Zeeman Effect for an Arbitrary Magnetic Field	261
14.2	Multielectron Atoms	264
14.2.1	Weak Magnetic Field	264
14.2.2	Strong Magnetic Field, the Paschen–Back Effect	266
14.3	The Stark Effect	266
14.3.1	Energy Shift of the Ground State	267
14.3.2	Excited States	267
	Problems	269
15.	Molecules	271
15.1	Qualitative Considerations	271
15.2	The Born–Oppenheimer Approximation	273
15.3	The Hydrogen Molecular Ion (H_2^+)	275
15.4	The Hydrogen Molecule H_2	278
15.5	Energy Levels of a Two-Atom Molecule: Vibrational and Rotational Levels	282
15.6	The van der Waals Force	284
	Problems	287
16.	Time Dependent Phenomena	289
16.1	The Heisenberg Picture for a Time Dependent Hamiltonian	289
16.2	The Sudden Approximation	291
16.3	Time Dependent Perturbation Theory	292
16.3.1	Perturbative Expansion	292
16.3.2	First-Order Transitions	294
16.3.3	Transitions into a Continuous Spectrum, the Golden Rule	294
16.3.4	Periodic Perturbations	297
16.4	Interaction with the Radiation Field	298
16.4.1	The Hamiltonian	298

16.4.2	Quantization of the Radiation Field	299
16.4.3	Spontaneous Emission	301
16.4.4	Electric Dipole ($E1$) Transitions	303
16.4.5	Selection Rules for Electric Dipole ($E1$) Transitions	303
16.4.6	The Lifetime for Electric Dipole Transitions	306
16.4.7	Electric Quadrupole and Magnetic Dipole Transitions	307
16.4.8	Absorption and Induced Emission	309
	Problems	310
17.	The Central Potential II	313
17.1	The Schrödinger Equation for a Spherically Symmetric Square Well	313
17.2	Spherical Bessel Functions	314
17.3	Bound States of the Spherical Potential Well	316
17.4	The Limiting Case of a Deep Potential Well	318
17.5	Continuum Solutions for the Potential Well	320
17.6	Expansion of Plane Waves in Spherical Harmonics	321
	Problems	324
18.	Scattering Theory	325
18.1	Scattering of a Wave Packet and Stationary States	325
18.1.1	The Wave Packet	325
18.1.2	Formal Solution of the Time Independent Schrödinger Equation	326
18.1.3	Asymptotic Behavior of the Wave Packet	328
18.2	The Scattering Cross Section	330
18.3	Partial Waves	331
18.4	The Optical Theorem	335
18.5	The Born Approximation	337
18.6	Inelastic Scattering	339
18.7	Scattering Phase Shifts	340
18.8	Resonance Scattering from a Potential Well	342
18.9	Low Energy s -Wave Scattering; the Scattering Length	346
18.10	Scattering at High Energies	349
18.11	Additional Remarks	351
18.11.1	Transformation to the Laboratory Frame	351
18.11.2	The Coulomb Potential	352
	Problems	352
19.	Supersymmetric Quantum Theory	355
19.1	Generalized Ladder Operators	355
19.2	Examples	358
19.2.1	Reflection-Free Potentials	358
19.2.2	The δ -function	361

19.2.3	The Harmonic Oscillator	361
19.2.4	The Coulomb Potential	362
19.3	Additional Remarks	365
	Problems	367
20.	State and Measurement in Quantum Mechanics	369
20.1	The Quantum Mechanical State, Causality, and Determinism	369
20.2	The Density Matrix	371
20.2.1	The Density Matrix for Pure and Mixed Ensembles	371
20.2.2	The von Neumann Equation	376
20.2.3	Spin-1/2 Systems	377
20.3	The Measurement Process	380
20.3.1	The Stern–Gerlach Experiment	380
20.3.2	The Quasiclassical Solution	381
20.3.3	The Stern–Gerlach Experiment as an Idealized Measurement	381
20.3.4	A General Experiment and Coupling to the Environment	383
20.3.5	Influence of an Observation on the Time Evolution .	387
20.3.6	Phase Relations in the Stern–Gerlach Experiment .	389
20.4	The EPR Argument, Hidden Variables, the Bell Inequality .	390
20.4.1	The EPR (Einstein–Podolsky–Rosen) Argument . .	390
20.4.2	The Bell Inequality	392
	Problems	396
Appendix	399
A.	Mathematical Tools for the Solution of Linear Differential Equations	399
A.1	The Fourier Transform	399
A.2	The Delta Function and Distributions	399
A.3	Green’s Functions	404
B.	Canonical and Kinetic Momentum	405
C.	Algebraic Determination of the Orbital Angular Momentum Eigenfunctions	406
D.	The Periodic Table and Important Physical Quantities	412
Subject Index	417