

Handbook of Financial Time Series

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Editors

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 Springer

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Foreword

The *Handbook of Financial Time Series*, edited by Andersen, Davis, Kreiss and Mikosch, is an impressive collection of survey articles by many of the leading contributors to the field. These articles are mostly very clearly written and present a sweep of the literature in a coherent pedagogical manner. The level of most of the contributions is mathematically sophisticated, and I imagine many of these chapters will find their way onto graduate reading lists in courses in financial economics and financial econometrics. In reading through these papers, I found many new insights and presentations even in areas that I know well.

The book is divided into five broad sections: GARCH-Modeling, Stochastic Volatility Modeling, Continuous Time Processes, Cointegration and Unit Roots, and Special Topics. These correspond generally to classes of stochastic processes that are applied in various finance contexts. However, there are other themes that cut across these classes. There are several papers that carefully articulate the probabilistic structure of these classes, while others are more focused on estimation. Still others derive properties of extremes for each class of processes, and evaluate persistence and the extent of long memory. Papers in many cases examine the stability of the process with tools to check for breaks and jumps. Finally there are applications to options, term structure, credit derivatives, risk management, microstructure models and other forecasting settings.

The GARCH family of models is nicely introduced by Teräsvirta and then the mathematical underpinning is elegantly and readably presented by Lindner with theorems on stationarity, ergodicity and tail behavior. In the same vein, Giraitis, Leipus and Surgailis examine the long memory properties of infinite order ARCH models with memory decay slower than GARCH, and Davis and Mikosch derive tail properties of GARCH models showing that they satisfy a power law and are in the maximum domain of attraction of the Fréchet distribution. The multivariate GARCH family is well surveyed by Silvennoinen and Teräsvirta. Linton and Čížek and Spokoiny, respectively, specify models which are non- or semi-parametric or which are only constant over intervals of homogeneity.

The section on Stochastic Volatility Modelling (SV) brings us up to date on the development of alternatives to GARCH style models. Davis and Mikosch in two chapters develop the somewhat easier underlying mathematical theory and tail properties of SV. They derive an important difference from GARCH models. While both stochastic volatility and GARCH processes exhibit volatility clustering, only the GARCH has clustering of extremes. Long memory is conveniently described by SV models in Hurvich and Soulier. Chib, Omori and Asai extend these analyses to multivariate systems although they do not envision very high dimensions. Estimation is covered in several chapters by Renault, Shephard and Andersen, and Jungbacker and Koopman.

The continuous time analysis begins with familiar Brownian motion processes and enhances them with jumps, dynamics, time deformation, correlation with returns and Lévy process innovations. Extreme value distributions are developed and estimation algorithms for discretely sampled processes are analyzed. Lindner discusses the idea of continuous time approximations to GARCH and SV models showing that the nature of the approximation must be carefully specified. The continuous time framework is then applied to several finance settings such as interest rate models by Björk, option pricing by Kallsen, and realized volatility by Andersen and Benzoni. The book then returns to analysis of first moments with surveys of discrete time models with unit roots, near unit roots, fractional unit roots and cointegration.

Finally, a remaining 13 chapters are collected in a section called Special Topics. These include very interesting chapters on copulas, non-parametric models, resampling methods, Markov switching models, structural break models and model selection. Patton and Sheppard examine univariate and multivariate volatility forecast comparisons. They show the advantages of a GLS correction, discuss multiple comparisons and economic loss functions. Bauwens and Hautsch survey a wide range of models for point processes that have been used in the finance literature to model arrival times of trades and quotes. The survey is well grounded in the statistical literature and the economics literature. Embrechts, Furrer and Kaufmann discuss different types of risk—market, credit, operational and insurance—and some of the leading approaches to estimation. Christoffersen applies the filtered historical simulation or FHS method to univariate and multivariate simulation based calculation of VaR, Expected Shortfall and active portfolio risks. Lando surveys the structural and reduced form approaches to modeling credit spreads. He focuses on CDS spreads and default dependence and gives a nice description of tests between contagion and factor structures in formulating dependence.

So make yourself a double cappuccino and relax in a comfortable chair, or adjust your headphones at 30,000 ft. over the Pacific, and dig in. There are treats in lots of different areas just waiting to be discovered.

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