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Control of Nonlinear Dynamical Systems

Methods and Applications

Springer

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Preface

This book is devoted to new methods of control for complex dynamical systems and deals with nonlinear control systems having several degrees of freedom, subjected to unknown disturbances, and containing uncertain parameters. Various constraints are imposed on control inputs and state variables or their combinations.

The book contains an introduction to the theory of optimal control and the theory of stability of motion, and also a description of some known methods based on these theories.

Major attention is given to new methods of control developed by the authors over the last 15 years. Mechanical and electromechanical systems described by nonlinear Lagrange's equations are considered. General methods are proposed for an effective construction of the required control, often in an explicit form. The book contains various techniques including the decomposition of nonlinear control systems with many degrees of freedom, piecewise linear feedback control based on Lyapunov's functions, methods which elaborate and extend the approaches of the conventional control theory, optimal control, differential games, and the theory of stability.

The distinctive feature of the methods developed in the book is that the controls obtained satisfy the imposed constraints and steer the dynamical system to a prescribed terminal state in finite time. Explicit upper estimates for the time of the process are given. In all cases, the control algorithms and the estimates obtained are strictly proven.

The methods are illustrated by a number of control problems for various engineering systems: robotic manipulators, pendular systems, electromechanical systems, electric motors, multibody systems with dry friction, etc. The efficiency of the proposed approaches is demonstrated by computer simulations.

The authors hope that the monograph will be a useful contribution to the scientific literature on the theory and methods of control for dynamical systems. The book could be of interest for scientists and engineers in the field of applied mathematics, mechanics, theory of control and its applications, and also for students and postgraduates.

Moscow,
April 2008

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Introduction

There exist numerous methods for the design of control for dynamical systems.

The classical methods of the theory of automatic control are meant for linear systems and represent the control in the form of a linear operator applied to the current phase state of the system. Shortcomings of this approach are obvious both in the vicinity of the prescribed terminal state as well as far from it. Near the terminal state, the magnitude of the control becomes small, so that control possibilities are not fully realized. As a result, the time of the control process occurs to be, strictly speaking, infinite, and the phase state can only tend asymptotically to the terminal state as time goes to infinity. On the other hand, far from the terminal state, the control magnitude becomes large and can violate the constraints usually imposed on the control. That is why it is difficult and often impossible to take account of the constraints imposed when the linear methods are used. Moreover, the classical methods based on linear models are usually inapplicable to nonlinear systems; at least, their applicability should be justified thoroughly.

In principle, the methods of the theory of optimal control can be applied to nonlinear systems. These methods take account of various constraints imposed on the control and, though with considerable complications, on the state variables. The methods of optimal control bring a dynamical system to a prescribed terminal state in an optimal (in a certain sense) way; for example, in a minimum time. However, to construct the optimal control for a nonlinear system is a very complicated problem, and its explicit solution is seldom available. Especially difficult is the construction of a feedback optimal control for a nonlinear system, even for a system with a small number of degrees of freedom and even with the help of modern computers.

There exist a number of other general methods of control: the method of systems with variable structure [123, 116, 115], the method of feedback linearization [70, 71, 91], and their various generalizations. However, these methods usually do not take into account constraints imposed on the control and state variables. Moreover, being very general, these methods do not take account of specific properties of mechanical systems such as conservation laws or the structure of basic equations of motions that can be presented in the Lagrangian or the Hamiltonian forms. Some other control methods applicable to nonlinear mechanical systems were developed in [61, 62, 94, 95, 59, 51, 52, 85, 90, 118].

In this book, some methods of control for nonlinear mechanical systems subjected to perturbations and uncertainties are proposed. These methods are applicable in the presence of various constraints on control and state variables. By taking into account some specific properties inherent in the equations of mechanical systems, these methods yield more efficient control algorithms compared with the methods developed for general systems of differential equations.

The authors' objective was to develop control methods having the following features.

1. Methods are applicable to nonlinear mechanical systems described by the Lagrange equations.

2. Methods are applicable to systems with many degrees of freedom.

3. Methods take into account the constraints imposed on the control, and, in a number of cases, also on the state variables as well as on both the control and state variables.

4. Methods bring the control system to the prescribed terminal state in finite time, and an efficient upper estimate is available for this time.

5. Methods are applicable in the presence of uncertain but bounded external perturbations and uncertain parameters of the system. Thus, the methods are robust.

6. There exist efficient algorithms for the construction of the desired feedback control.

7. Efficient sufficient controllability conditions are stated for the control methods proposed.

8. In all cases, a rigorous mathematical justification of the proposed methods is given.

It is clear that the above requirements are very useful and important from the standpoint of the control theory as well as various practical applications.

Several methods are proposed and developed in the book, and not all of them possess all of the features 1–8 listed above. Properties 3, 4, 7, and 8 are always fulfilled, whereas other features are inherent in some of the methods and not present in others.

The book consists of 10 chapters.

Chapters 2, 3, 5, and 6 deal with nonlinear mechanical systems with many degrees of freedom governed by Lagrange's equations and subjected to control and perturbation forces.

These equations are taken in the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} = U_i + Q_i, \quad i = 1, \ldots, n. \tag{0.1}$$

Here, t is time, the dots denote time derivatives, q_i are the generalized coordinates, $\dot{q}_i$ are the generalized velocities, U_i are the generalized control forces, Q_i are all other generalized forces including uncertain perturbations, n is the number of degrees of freedom, and $T(q, \dot{q})$ is the kinetic energy of the system. The kinetic energy is a symmetric positive definite quadratic form of the generalized velocities $\dot{q}_i$:

$$T(q,\dot{q}) = \frac{1}{2}\langle A(q)\dot{q},\dot{q}\rangle = \frac{1}{2}\sum_{j,k=1}^{n} a_{jk}(q)\dot{q}_j\dot{q}_k. \tag{0.2}$$

Here, q and $\dot{q}$ are the n-vectors of the generalized coordinates and velocities, respectively, and the brackets $\langle\cdot,\cdot\rangle$ denote the scalar product of vectors.

The quadratic form (0.2) satisfies the conditions

$$m|\dot{q}|^2 \le \langle A(q)\dot{q},\dot{q}\rangle \le M|\dot{q}|^2 \tag{0.3}$$

for any $q \in R^n$ and $\dot{q} \in R^n$, where m and M are positive constants such that $M > m$. Condition (0.3) implies that all eigenvalues of the matrix $A(q)$, for all $q \in R^n$, belong to the interval $[m,M]$.

In Chapters 2 and 3, the coefficients of the quadratic form (0.2) are supposed to be known functions of the coordinates: $a_{jk} = a_{jk}(q)$. In Chapters 5 and 6, the functions $a_{jk}(q)$ may be unknown but the constants m and M in (0.3) are given. Also, the case of rheonomic systems for which $T = T(q,\dot{q},t)$ is considered in Chapter 5.

We suppose that the control forces are subjected to the geometric constraints at any time instant:

$$|U_i| \le U_i^0, \qquad i = 1,\dots,n, \tag{0.4}$$

where U_i^0 are given constants.

The generalized forces Q_i may be more or less arbitrary functions of the coordinates, velocities, and time; these functions may be unknown but are assumed bounded by the inequality

$$|Q_i(q,\dot{q},t)| \le Q_i^0, \qquad i = 1,\dots,n, \tag{0.5}$$

The constants Q_i^0 are supposed to be known, and certain upper bounds are imposed on Q_i^0 in order to achieve the control objective.

The control problem is formulated as follows:

Problem 0.1. It is required to construct the feedback control $U_i(q,\dot{q})$ that brings system (0.1) subject to constraints (0.3)–(0.5) from the given initial state

$$q(t_0) = q^0, \qquad \dot{q}(t_0) = \dot{q}^0 \tag{0.6}$$

at a given initial time instant $t = t_0$ to the prescribed terminal state with zero terminal generalized velocities

$$q(t_*) = q^*, \qquad \dot{q}(t_*) = 0 \tag{0.7}$$

in finite time. The time instant t_* is not prescribed but an upper estimate on it should be obtained.

In some sections of Chapter 3, the case of nonzero terminal velocities $\dot{q}_i(t_*) \neq 0$ is also considered.

In many practical applications, it is desirable to bring the system from the state (0.6) to the state (0.7) as soon as possible, i.e., to minimize t_*. However, to construct the exact solution of this time-optimal control problem for the nonlinear system is a very difficult problem, especially, if one desires to obtain the feedback control. The methods proposed in Chapters 2, 3, and 5–8 do not provide the time-optimal control but include certain procedures of optimization of the time t_*. Therefore, these methods may sometimes be called suboptimal.

The main difficulties arising in the construction of the control for system (0.1) are due to its nonlinearity and its high order. The complex nonlinear dynamical interaction of different degrees of freedom of the system is characterized by the elements $a_{jk}(q)$ of the matrix $A(q)$ of the kinetic energy. Another property that complicates the construction of the control is the fact that the dimension n of the control vector is two times less than the order of system (0.1).

Manipulation robots can be regarded as typical examples of mechanical or electromechanical systems described by equations (0.1). Being an essential part of automated manufacturing systems, these robots can serve for various technological operations. A manipulation robot is a controlled mechanical system that consist of one or several manipulators, a control system, drives (actuators), and grippers. A manipulator can perform a wide range of three-dimensional motions and bring objects (instruments and/or workpieces) to a prescribed position and orientation in space. Various types of drives, namely, electric, hydraulic, pneumatic, and other, are employed in robotic manipulators, the electric drives being the most widespread.

The manipulator is a multibody system that consists of several links connected by joints. The drives are usually located at the joints or inside links adjacent to the joints. Relative angular or linear displacements of neighboring links are usually chosen as the generalized coordinates q_i of the manipulator. The kinetic energy $T(q,\dot{q})$ of the manipulator consists of the kinetic energy of its links and also, if the drives are taken into account, the kinetic energy of electric drives and gears. The Lagrange equations (0.1) of the manipulator involve the generalized forces Q_i due to the weight and resistance forces; the latter are often not known exactly and may change during operations. Moreover, parameters of the manipulator may also change in an unpredictable way. Therefore, some of the forces Q_i should be regarded as uncertain perturbations. The control forces U_i are forces and/or torques produced by the drives.

Since the manipulator is a nonlinear multibody system subject to uncertain perturbations, it is quite natural to consider the problem of control for the manipulator as a nonlinear control problem formulated above as Problem 0.1.

Let us outline briefly the contents of Chapters 1–10.

Chapter 1 gives an introduction to the theory of optimal control. Basic concepts and results of this theory, especially the Pontryagin maximum principle, are often used throughout the book. The maximum principle is formulated and illustrated by several examples. The feedback optimal controls obtained for these examples are often referred to in the following chapters.

In Chapters 2 and 3, the methods of decomposition for Problem 0.1 are proposed and developed. The essence of these methods is a transformation of the original nonlinear system (0.1) with n degrees of freedom to the set of n independent linear subsystems

$$\ddot{x}_i = u_i + v_i, \quad i = 1, \ldots, n. \tag{0.8}$$

Here, x_i are the new (transformed) generalized coordinates, u_i are the new controls, and forces v_i include the generalized forces Q_i, as well as the nonlinear terms that describe the interaction of different degrees of freedom in system (0.1). The perturbations v_i in system (0.8) are treated as uncertain but bounded forces; they can also be regarded as the controls of another player that counteract the controls u_i.

The original constraints (0.3)–(0.5) imposed on the kinetic energy and generalized forces of system (0.1) are, under certain conditions, reduced to the following normalized constraints on controls u_i and disturbances v_i:

$$|u_i| \leq 1, \quad |v_i| \leq \rho_i, \quad \rho_i < 1, \quad i = 1, \ldots, n. \tag{0.9}$$

By applying the approach of differential games [69, 79] to system (0.8) subject to constraints (0.9), we obtain the feedback control $u_i(x_i, \dot{x}_i)$ that solves the control problem for the ith subsystem, if $\rho_i < 1$.

Besides the game-theoretical technique, a simpler approach to the control construction is also considered, where the perturbations in system (0.8) are completely ignored. As the control $u_i(x_i, \dot{x}_i)$ of the ith subsystem (0.8) we choose the time-optimal feedback control for the system

$$\ddot{x}_i = u_i, \quad i = 1, \ldots, n.$$

It is shown that this simplified approach is effective, i.e., brings the ith subsystem (0.8) to the prescribed terminal state, if and only if the number ρ_i in (0.9) does not exceed the golden section ratio:

$$\rho_i < \rho^* = \frac{1}{2}(\sqrt{5} - 1) \approx 0.618.$$

In other words, uncertain but bounded perturbations can be neglected while constructing the feedback control, if and only if their magnitude divided by the magnitude of the control does not exceed the golden section ratio ρ^*.

Two versions of the decomposition method presented in Chapters 2 and 3 differ both by the assumptions made and the results obtained.

The assumptions of the second version (Chapter 3) are less restrictive; on the other hand, the time of the control process is usually less for the first version (Chapter 2).

As a result of each decomposition method, explicit feedback control laws for the original system (0.1) are obtained. These control laws $U_i = U_i(q, \dot{q})$, $i = 1, \ldots, n$, satisfy the imposed constraints (0.4) and bring the system to the terminal state (0.7) under any admissible perturbations $Q_i(q, \dot{q}, t)$ subject to conditions (0.5). Sufficient controllability conditions are derived for the methods proposed. The time of control t_* is finite, and explicit upper bounds on t_* are given.

Certain generalizations and modifications of the decomposition methods are presented in Chapters 2 and 3. The original system (0.1) with n degrees of freedom can be reduced to sets of subsystems more complicated than (0.8); these subsystems can be either linear or nonlinear, and these two cases are examined. The decomposition method is extended to the case of nonzero prescribed terminal velocity $\dot{q}_i(t_*)$ in (0.7), and also to the problem of tracking the prescribed trajectory of motion.

Control problems for the manipulation robots with several degrees of freedom are considered as examples illustrating the methods proposed. Purely mechanical models of robots as well as electromechanical models that take account of processes in electric circuits are considered.

Chapter 4 briefly presents basic concepts and results of the theory of stability. Here, the notion of the Lyapunov function plays the central role, and the corresponding theorems using this notion are formulated. The Lyapunov functions are widely used in the following Chapters 5 and 6.

In these chapters, the method of control based on the piecewise linear feedback for system (0.1)–(0.7) is presented. The required control vector U is sought in the form

$$U = -\beta(q - q^*) - \alpha\dot{q}, \quad U = (U_1, \ldots, U_n), \tag{0.10}$$

where α and β are scalar coefficients.

During the motion, the coefficients increase in accordance with a certain algorithm and may tend to infinity as the system approaches the terminal state (0.7), i.e., $t \to t_*$. However, the control forces (0.10) stay bounded and satisfy the imposed constraints (0.4).

In Chapter 5, the coefficients α and β are piecewise constant functions of time. These coefficients change when the system reaches certain prescribed ellipsoidal surfaces in $2n$-dimensional phase space. In Chapter 6, the coefficients α and β are continuous functions of time.

In both Chapters 5 and 6, the proposed algorithms are rigorously justified with the help of the second Lyapunov method. It is proven that this control technique brings the system (0.1) to the prescribed terminal state (0.7) in finite time. An explicit upper bound for this time is obtained.

The methods of Chapters 5 and 6 are applicable not only in the case of uncertain perturbations satisfying (0.5), but also if the matrix A of the kinetic energy (0.2) is uncertain. It is only necessary that restrictions (0.3) hold and the constants m and M be known.

The approach based on the feedback control (0.10) is extended also to rheonomic systems whose kinetic energy is a second-order polynomial of the generalized velocities with coefficients depending explicitly on the generalized coordinates and time (Chapter 5). The coefficients of the kinetic energy are assumed unknown, and the system is acted upon by uncertain perturbations. The control algorithm is given that brings the rheonomic system to the prescribed terminal state by a bounded control force.

Several examples of controlled multibody systems are considered in Chapters 5 and 6. Some parameters of the systems, namely, masses, coefficients of stiffness and friction, are assumed unknown, and uncertain perturbations are also taken into account. It is shown that the methods proposed in Chapters 5 and 6 can control such systems and bring them to the terminal state; moreover, the methods are efficient even if the sufficient controllability conditions derived in Chapters 5 and 6 are not satisfied.

Note that, together with the methods discussed in the book, there are other approaches that ensure asymptotic stability of a given state of the system, i.e., bring the system to this state as $t \to \infty$. In practice, one needs to bring the system to the vicinity of the prescribed state; therefore, the algorithms based on the asymptotic stability practically solve the control problem in finite time. However, as the required vicinity of the terminal state decreases and tends to zero, the time of motion for the control methods ensuring the asymptotic stability increases and tends to infinity. By contrast, the methods proposed in this book ensure that the time of motion is finite, and explicit upper bounds for this time are given in Chapters 2, 3, 5, and 6.

In Chapters 1–6, systems with finitely many degrees of freedom are considered; these are described by systems of ordinary differential equations. A number of books and papers (see, for example, [25, 122, 86, 113, 117, 87]) are devoted to control problems for systems with distributed parameters that are described by partial differential equations. The methods of decomposition proposed in Chapters 2 and 3 can also be applied to systems with distributed parameters.

In Chapter 7, control systems with distributed parameters are considered. These systems are described by linear partial differential equations resolved with respect to the first or the second time derivative. The first case corresponds, for example, to the heat equation, and the second to the wave equation. The control is supposed to be distributed and bounded; it is described by the corresponding terms in the right-hand side of the equation. The control problem is to bring the system to the zero terminal state in finite time. The proposed control method is based on the decomposition of the original system into subsystems with the help of the Fourier method. After that, the time-optimal feedback control is applied to each mode. A peculiarity of this control problem is that there is an infinite (countable) number of modes.

Sufficient controllability conditions are derived. The required feedback control is obtained, together with upper estimates for the time of the control process. These results are illustrated by examples.

In Chapters 8–10, we return to control systems governed by ordinary differential equations.

In Chapter 8, we consider linear systems subject to various constraints. Control and phase constraints, as well as mixed constraints imposed on both the control and the state variables are considered. Integral constraints on control and state variables are also taken into account. Though the original systems are linear, the presence of complex constraints makes the control problem essentially nonlinear and rather complicated.

Note that various constraints on control and state variables are often encountered in applications. For example, if the system includes an electric drive, it is usually necessary to take account of constraints on the angular velocity of the shaft, the control torque, and also on their combination. Integral constraints typically occur, if there are energy restrictions.

The approach developed in Chapter 8 is a generalization of the well-known Kalman's method [72, 73]. This method, originally proposed for the control of linear systems in the absence of constraints, is based on the representation of the control as a linear combination of the eigenmodes of motion. In Chapter 8, this method is extended to some cases with different constraints. Explicit control laws are obtained for various oscillatory systems, in particular, a system of many oscillators controlled by one bounded control. For certain systems of the second order, the controls obtained are compared with time-optimal controls. The method is applied also to systems of the fourth (and higher) order with mixed constraints. The models considered here correspond to mechanical and electromechanical systems containing an oscillator and an electric motor. Sufficient controllability conditions derived in Chapter 8 ensure that the control obtained brings the system to the prescribed state in finite time, and all mixed constraints are satisfied.

Chapter 9 is devoted to several control problems for a simple dynamical system with one degree of freedom described by the second Newton's law and subject to different constraints that model real constraints typical for actuators. The system is to be brought to the origin of the coordinate system in the phase plane.

First, the time-optimal control problem is considered in the presence of mixed constraints imposed on the control and state variables. The time-optimal feedback control is obtained. As an example, a control problem for the electric drive is examined.

Next, a constraint is imposed on the rate of change of the control force. Such a constraint is often inherent in various drives. The resultant equations are reduced to a third-order system. The time-optimal control problem for this system is solved, and the required control is obtained in the open-loop as well as in the feedback form. The solution of this problem is based on a group-invariant approach that reduces the number of the essential phase variables from three to two.

At the end of Chapter 9, it is supposed that the absolute value of the control force can grow only gradually, with a bounded rate, whereas this force can be switched off instantly. Under these assumptions, which model real drives, we find the control that brings the system to a prescribed state and has the simplest possible structure.

In Chapter 10, two time-optimal control problems for the nonlinear pendulum are solved. The pendulum is a classical nonlinear system that often serves as a test model in nonlinear dynamics and control theory. We assume that the bounded control torque is applied to the axis of the pendulum. The terminal state is either the upper unstable or the lower stable equilibrium position of the pendulum; thus, we study the time-optimal swing-up and damping control problems, respectively. The peculiarity of these problems is that the pendulum has a cylindrical phase space and an infinite number of equivalent equilibrium positions which differ by 2π. The feedback controls for both the swing-up and the dumping cases have a very complicated structure, which is obtained numerically for a wide range of the system parameters.

Thus, a number of new methods for the control of nonlinear dynamical systems are presented in the book. The control algorithms are described, their rigorous mathematical proof is given, and a number of specific control problems are analyzed and solved by these methods.

This book is mostly based on the results obtained by the authors during the last two decades.