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Bernt Øksendal · Agnès Sulem

# **Applied Stochastic Control of Jump Diffusions**

2<sup>nd</sup> Edition

With 27 Figures

 Springer

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To my family

Eva, Elise, Anders, and Karina

B. Ø.

A tous ceux qui m'accompagnent

A. S.

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## Preface to the Second Edition

In this second edition, we have added a chapter on optimal control of random jump fields (solutions of stochastic *partial* differential equations) and partial information control (Chap. 10). We have also added a section on optimal stopping with delayed information (Sect. 2.3). It has always been our intention to give a contemporary presentation of applied stochastic control, and we hope that the addition of these recent developments will contribute in this direction.

We have also made a number of corrections and other improvements, many of them based on helpful comments from our readers. In particular, we would like to thank Andreas Kyprianou for his valuable communications. We are also grateful to (in alphabetical order) Knut Aase, Jean-Philippe Chancelier, Inga Eide, Emil Framnes, Arne-Christian Lund, Jose-Luis Menaldi, Tamás K. Papp, Atle Seierstad, and Jens Arne Sukkestad for pointing out errors and suggesting improvements. Our special thanks go to Martine Verneuille for her skillful typing.

Oslo and Paris, November 2006

Bernt Øksendal and Agnès Sulem

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## Preface of the First Edition

Jump diffusions are solutions of stochastic differential equations driven by Lévy processes. Since a Lévy process  $\eta(t)$  can be written as a linear combination of  $t$ , a Brownian motion  $B(t)$  and a pure jump process, jump diffusions represent a natural and useful generalization of Itô diffusions. They have received a lot of attention in the last years because of their many applications, particularly in economics.

There exist today several excellent monographs on Lévy processes. However, very few of them – if any – discuss the optimal control, optimal stopping, and impulse control of the corresponding jump diffusions, which is the subject of this book. Moreover, our presentation differs from these books in that it emphasizes the applied aspect of the theory. Therefore, we focus mostly on useful verification theorems and we illustrate the use of the theory by giving examples and exercises throughout the text. Detailed solutions of some of the exercises are given at the end of the book. The exercises to which a solution is provided, are marked with an asterix \*. It is our hope that this book will fill a gap in the literature and that it will be a useful text for students, researchers, and practitioners in stochastic analysis and its many applications. Although most of our results are motivated by examples in economics and finance, the results are general and can be applied in a wide variety of situations. To emphasize this, we have also included examples in biology and physics/engineering.

This book is partially based on courses given at the Norwegian School of Economics and Business Administration (NHH) in Bergen, Norway, during the Spring semesters 2000 and 2002, at INSEA in Rabat, Morocco in September 2000, at Odense University in August 2001 and at ENSAE in Paris in February 2002.

Oslo and Paris, August 2004

Bernt Øksendal and Agnès Sulem

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Oslo and Paris, August 2004

Bernt Øksendal and Agnès Sulem

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