

Foundations of Engineering Mechanics

V. I. Babitsky, Theory of Vibro-Impact Systems and Applications

Springer-Verlag
Berlin Heidelberg GmbH

V. I. Babitsky

Theory of Vibro-Impact Systems and Applications

Translated by N. Birkett

With 146 Figures



Springer

Series Editors:

Prof. Dr V. I. Babitsky
Loughborough University
Department of Mechanical Engineering
SE11 3TU Loughborough, Leicestershire
UK

Prof. Dr J. Wittenburg
Universität Karlsruhe (TH)
Institut für Mechanik
Kaiserstraße 12
D-76128 Karlsruhe/Germany

Author:

Prof. Dr V. I. Babitsky

Translator:

Nicolas Birkett
Grangefield Drive 28
LE7 7NB Rothley, Leicester
UK

ISBN 978-3-662-22534-9 ISBN 978-3-540-69635-3 (eBook)
DOI 10.1007/978-3-540-69635-3

Cataloging-in-Publication Data applied for
Die Deutsche Bibliothek – CIP Einheitsaufnahme

Babickij, Vladimir I.:

Theory of vibro-impact systems with applications / V. I. Babitsky.

Transl. by N. Birkett. –Berlin; Heidelberg; New York; Barcelona; Budapest; Hong Kong; London;
Milan; Paris; Singapore; Tokyo:

Springer, 1998

(Foundations of engineering mechanics)

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in other ways, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution act under German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1998

Originally published by Springer-Verlag Berlin Heidelberg New York in 1998.

Softcover reprint of the hardcover 1st edition 1998

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Product liability: The publisher cannot guarantee the accuracy of any information about dosage and application contained in this book. In every individual case the user must check such information by consulting the relevant literature.

Cover design: de`blik, Berlin

Typesetting: Camera ready by author

SPIN: 10630548

62/3020 - 5 4 3 2 1 0 - Printed on acid-free paper

To Eleonora and Ilya

Preface to the English edition

The drop of rain maketh a hole in the stone, not by violence, but by oft falling.

Bishop Hugh Latimer

(Born 1485 in Leicestershire, was burned in 1555 for heresy at Oxford)

The first edition of this book, published in Russian, generated wide interest in the problem within Russia but was not widely known outside. The aim of the author in writing this book on the synergistic effects of multiple impacts was to achieve the following:

- to understand the unusual resonant behaviour of vibro-impact systems,
- to accommodate vibro-impact systems within the general theory of nonlinear vibrations for all forms of mechanical model and excitations,
- to compare different mathematical methods for the analysis of vibro-impact systems and to develop effective approximate methods based on frequency and structural approaches,
- to apply frequency methods for analysis of vibro-impact processes in complex mechanical structures interacting with driving and control units,
- to develop an approach for the structural synthesis of vibro-impact systems,
- to explain the dynamics of some important engineering systems.

In order to solve these problems, the author extended the mechanical description of vibro-impact systems by applying contact compliances, distributed flexible mechanical structures and complex excitations.

The theory of vibro-impact systems that was developed, revealed new nonlinear phenomena. The exploitation of these phenomena gave an impetus for the promotion of different engineering applications. To achieve this the author founded the Laboratory at the Blagonravov Institute of Machine Studies of the Russian Academy of Sciences which carried out research and development on a wide range of engineering systems that resulted in the design of effective rigs. These activities were generally described in the article included as Appendix II of this book, which we recommend to the reader for initial acquaintance with the problem. Regular Symposiums and Seminars on “Dynamics of Vibro-Impact Systems” were conducted within the mentioned activity, which stimulated intensive development of this area and mutual applications.

The following lectures and demonstrations in different countries together with

interest in this topic from industry, inspired the author to propose the revised English translation of this book. The development in recent years of powerful software such as Matlab-Simulink gave new opportunities to simulate vibro-impact structures using frequency and structural methods. The book can also be used as a foundation for these new applications. In order to demonstrate these possibilities, we have added to the book Appendix I, written together with Dr A Veprik who also helped the author with the description of recently discovered vibro-impact phenomena in §14 section 5 and §16 section 4. The book is further upgraded with an additional list of new references.

We hope that the book will introduce these general approaches for important areas of mechanical engineering to English speaking readers and help the generation of new research and development specialists in the analysis of such complex mechanical structures. From the book, students and their teachers can acquire knowledge of the mechanical and engineering aspects of dynamics of strongly nonlinear systems.

The translator of the book, Mr Birkett, and the staff of Springer-Verlag were extremely co-operative during all stages in the preparation of the book. The author is very grateful to all the individuals who participated in accomplishing this project.

Vladimir Babitsky

Loughborough, Leicestershire

Preface to the Russian edition

It is difficult to think of a physical effect other than impact that generates more significant results for so small a resource requirement. It is constantly attracts interest to study it and its varied applications. The use of impact principles in household and industrial practices has evolved on its own over the years. For thousands of years, impact tools have developed in a complex manner, remaining one of the most productive and effective tools throughout. From the first primitive hammers, to contemporary forging and riveting machines, from ancient pick-axes to novel types of hammers and pneumatic drills, from battering rams to perfectly designed concrete breakers and perforators, these represent the extremes of this long lasting process.

From the wide circle of engineering problems linked to the study of impact interactions, it is reasonable to separate the class of problems where *systematic* collisions occur. Systems in which such a form of interaction occurs are known as *vibro-impact*. This type of interaction becomes significant compared to a single impact only when the collision frequency matches the natural frequency of the interacting elastic systems. In the state of affairs specified, collisions occurring in the excited system, which has some energy resource that can be built up from impact to impact, cause various forms of vibration effects. What has been said defines the subject of the *theory of vibro-impact systems* as a self-contained section within the theory of mechanical vibrations.

The book systematically presents the theory of vibration for mechanical systems whose motion is accompanied by collisions between elements. Loss of kinematic links within machines and collisions between the elements of kinematic pairs and structures cause the establishment of intensive vibro-impact processes, as the result of which the tuning of elastic systems changes significantly, as do their natural frequency distributions and the character of transient processes, and a variety of nonlinear effects can arise. Thus the study of problems in dynamics in terms of accuracy, stability and vibration activity, requires vibro-impact processes to be considered in systems with various structural forms, having both distributed and lumped masses, nonlinearities and being exposed to the effects of a variety of regular and random forces.

In addition, vibro-impact regimes form the basic working process for a broad class of machines, drives and installations with the widest functional purposes. To this class belong machines for driving and extracting piles, tamping and cultivating ground, crushing and grinding materials, compacting concrete and foundry mixtures, vibro-impact test devices, various examples of drill, percussion devices, riveters, nut-tightening machines, etc. For this class of system, the characteristic periodic motion arises either due to forced vibration or self-

sustaining vibration.

The range of constructional variations for vibro-impact mechanisms and systems requires the development of methods not only to solve the problems of analysis and tuning, but also the development of methods to synthesise dynamic structures which best fulfil the optimum form of motion.

Nowadays, the basic techniques for the theoretical study of vibro-impact processes are the exact methods of nonlinear mechanics, based on stitching solutions describing adjacent intervals of motion, which are separated by the instants of impact. These methods allowed a detailed study of the complicated dynamics picture for the motion of a series of systems and revealed a number of fundamental properties inherent to them. Results obtained in this way are reflected to a considerable extent in the monograph by A E and A A Kobrinsky [118]*.

Searching for solutions by exact methods is a very tedious procedure, which limits the scope of their application. It is particularly difficult to use them as the system dimensions increase in number, when it becomes necessary to take into account additional nonlinear factors and the complicated character of excitation from the effects of non-periodic and random forces.

In addition, to solve a large circle of problems in the theory of vibro-impact systems, and generally in the theory of nonlinear vibrations, it is unnecessary to define motion in a detailed manner. Foremost in this context, are problems connected with the *tuning* of such systems: finding their resonance conditions, the number and type of resonance bands, studying questions of stability and self-excitation etc. All these effects already appear when the basic harmonic components of vibration are analysed, since one of the effective methods of studying nonlinear systems is to separate from the general mathematical description the simpler relationships which characterise most of the corresponding patterns of vibration. These ideas, which come from the general procedure for the asymptotic presentation of solutions as power series of small parameters, have shown themselves to be particularly effective when transformed to spectral representation and the ideas of equivalent linearisation.

The first reported connection was established by N M Krylov and N N Bogolyubov, who provided the first approximation for the asymptotic method they developed in the form of *equivalent (harmonic) linearisation*. Later the harmonic linearisation method was widely developed and applied in the works of V A Kotelnikov, L S Goldfarb, W Oppelt, R Kochenburger, J D Loeb, E P Popov, K Magnus, L A Zadeh, and many other investigators, principally being applied to problems in the theory of automatic control. The contemporary state of this question and a bibliography appear in a series of collective monographs [162, 163, 173] published under the general editorship of E P Popov.

In the 1950s, P Booton [66] and I E Kazakov [112] independently presented an effective form of equivalent linearisation for the study of nonlinear *stochastic* processes, which is known as the *statistical linearisation* method. Techniques for applying the method were developed in the books by I E Kazakov and B G Dostupov [113] and A A Pervozvansky [177]. Finally we must mention the suggestions of M Z Kolovsky and A A Pervozvansky for the original form of

* See also Chapters 7 to 9 in [117] written by R E Brunstein and A E Kobrinsky.

equivalent linearisation by *distribution function* [128, 130] which made it possible to study multiharmonic processes in nonlinear systems. This method and its application to problems in the theory of nonlinear vibration protection is described in the monograph by M Z Kolovsky [111].

Systematic application of the range of methods described for equivalent linearisation and the solution of problems in the theory of vibro-impact systems forms the basis of the material in this book. In this way it was possible to consider the basic forms of vibration motion for vibro-impact systems of arbitrary dimensions in a unified manner using simple methods and to develop a sufficiently general theory for tuning a wide range of systems, such as impact vibration absorbers, pneumatic impact machines, devices for ultrasonic machining of materials, etc., and to reveal new nonlinear effects.

Sometimes in the analysis of a range of problems in the theory of vibro-impact systems, information yielded by equivalent linearisation methods is insufficient. This happens when it is necessary to obtain an idea of the law of motion or the trajectory of the system elements, for example, when estimating collision velocities, studying the spectral characteristics of motion, defining existence domains of dynamic models in configurational space, etc. For all of these, methods are suggested, based on full approximations of the laws of motion for vibro-impact systems with the help of so-called *impulse-frequency characteristics*, widely studied in the works of E N Rosenvasser [191, 192,]. Following the traditions of mechanics these characteristics will be known as *periodic Green's functions* in this book

Together with these, procedures for exact methods are presented and applied, particularly in those cases when they can help the understanding of the nature and accuracy of approximate methods, but also for the better understanding of the effects studied. All the methods described and applied in this book do not assume that the reader has any previous knowledge of them.

The book consists of five chapters. The first chapter presents a short summary of contemporary ideas on the theory of collisions between solids and gives examples of simple vibration systems showing exact methods of analysis for the basic nonlinear behaviour of these systems in the presence of collisions. The character of resonance phenomena in vibro-impact systems and dynamics of the basic structure of vibro-impact machines are considered.

The second chapter introduces equations describing the process of vibro-impact interaction in general types of system undergoing arbitrary excitation and deals with the analysis of frequency characteristics for vibro-impact systems and the processes they cause. The concept of equivalent linearisation of vibro-impact systems is explained and harmonic and statistical linearisation coefficients for basic types of link between colliding elements are calculated.

The third chapter gives a systematic presentation of vibro-impact processes in systems with one degree of freedom based on approximate methods. Dynamics, stability, transient processes of periodic force vibration, parametric vibration and self-sustaining vibration are considered. Systems with random excitation are also studied. Special attention is given to sub-periodic regimes, and effective techniques for calculating these are developed. Throughout the explanations, descriptions of corresponding calculation methods are given and amplitude-

frequency characteristics are revealed and analysed, and a comparison of approximate and exact solutions is developed. The influence of the characteristics of the source of excitation on vibro-impact processes is estimated.

The fourth chapter develops analytical methods extended to systems with arbitrary degrees of freedom and is applied to the study of vibro-impact processes occurring during axial vibration of rods and bending vibration of beams for the analysis of systems with impact dynamic absorbers.

Finally, the fifth chapter presents problems in the structural synthesis of self-sustaining vibro-impact systems. Exact solutions are presented as a series of problems based on optimal control methods, and approximate methods to find the structure of controllers for general types of system are given using harmonic linearisation. These methods are applied to the solution of problems of excitation and stabilisation of autoresonant regimes in vibro-impact systems.

Trying to reflect the qualitative features of vibro-impact systems as objectively as possible, we have tried everywhere to develop the results up to the final analytical expressions, giving clear physical interpretations. In addition, experimental data is presented to confirm the basic effects.

The book is designed for engineers and researchers immersed in the study and design of systems involving mechanical vibrations, and also for lecturers and students interested in nonlinear vibrations. To study the book a reasonable knowledge of engineering based general mathematics will be adequate, covering elements of Fourier and Laplace transform theory and probability theory, together with a knowledge of basic theory of mechanical vibrations and stability of motion, as presented in the book by I M Babakov [24]*. All additional mathematical information is presented in the text during the explanation with references to the source where a detailed explanation is given.

The concluding bibliography lists only works either mentioned or used in the text, together with a series of related works of a general character.

Those wishing to use approximate methods immediately can begin their study with Chapter 2. To a considerable extent, Chapter 5 can be read as stand-alone. A self-contained concentrated explanation is also given for the statistical aspects of vibro-impact theory, to be found in §8, §13 and partially in §14 section 6 and §17 section 4. Before beginning to study these topics it is better to read through §6 first.

A E Kobrinsky first raised the author's interest in vibro-impact systems. The original ideas for the approximate methods developed in this book started as a series of papers completed in conjunction with M Z Kolovsky; whose advice and constant support was given freely to the author both during the planning of this book and during the writing process. V O Kononenko and Y G Panovko endorsed the plan of the book, and P F Nagaev made kind comments when the book was in manuscript form. V K Astashev helped greatly and §1 section 7, §2, §4, §15 were written with his help. The deep interest in the methods developed, shown by my colleagues in the Institute of Machine Studies such as T S Akinfiev, B A

*Translator's Note. For texts in English covering similar material the reader should consult W T Thomson's "Theory of vibrations with applications", 4th edition, (Chapman and Hall) or S S Rao's "Mechanical vibrations", 3rd edition (Addison-Wesley).

Borovkov, M E Hertz. M Y Israilovich, V L Krupenin and A N Tresviatsky, who were concerned in the development of a series of problems and in discussion of the results, was very stimulating for the author. The author sincerely thanks all the people named.

V Babitsky

Moscow

Contents

Preface to English edition.....	vii
Preface to Russian edition	ix
Contents	xv

Chapter 1 **Vibro-impact systems**

§1. Collision of solids.....	1
§2. Oscillator with fixed stop	20
§3. Oscillator with symmetrical stops.....	37
§4. Principles of percussion machine dynamics	53
§5. Stitching method.....	69

Chapter 2 **Equivalent linearisation of vibro-impact systems**

§6. Equations of vibro-impact systems.....	75
§7. Harmonic linearisation of vibro-impact systems	94
§8. Statistical linearisation of vibro-impact systems.....	108

Chapter 3 **Vibro-impact processes in systems with single degree of freedom**

§9. Forced vibration	125
§10. Stability and transients of forced vibration.....	140
§11. Parametric vibration	148
§12. Vibration of self-sustaining systems	165
§13. Random vibration.....	185

Chapter 4 **Vibro-impact processes in systems with multiple degrees of freedom**

§14. Vibro-impact interactions of elastic systems	193
§15. Vibro-impact interactions of visco-elastic bars	212
§16. Vibro-impact regimes in vibrating beams.....	229
§17. Theory of impact dynamic absorber	243

Chapter 5 Structural synthesis of self-sustaining vibro-impact systems

§18. Optimal periodic motions of vibro-impact systems 251
§19. Synthesis of self-sustaining systems 260
§20. Synthesis of autoresonant systems 271

Bibliography 279
Additional Bibliography 287

Index 292

Appendix I AI-1
Appendix II AII-1