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Stanisław Gawiejnowicz

# Time-Dependent Scheduling

with 26 Figures and 48 Tables

 Springer

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*To my parents*

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## Preface

The book presented to the reader is devoted to time-dependent scheduling. Scheduling problems, in general, consist in the allocation of resources over time in order to perform a set of jobs. Any allocation that meets all requirements concerning the jobs and resources is called a feasible schedule. The quality of a schedule is measured by a criterion function. The aim of scheduling is to find, among all feasible schedules, a schedule that optimizes the criterion function. A solution to an arbitrary scheduling problem consists in giving a polynomial-time algorithm generating either an optimal schedule or a schedule that is close to the optimal one, if the given scheduling problem has been proved to be computationally intractable. The scheduling problems are subject of interest of the scheduling theory, originated in mid-fifties of the twentieth century. The theory has been developing dynamically and new research areas constantly come into existence. The subject of this book, time-dependent scheduling, is one of such areas.

In time-dependent scheduling, the processing time of a job is variable and depends on the starting time of the job. This crucial assumption allows us to apply the scheduling theory to a broader spectrum of problems. For example, in the framework of the time-dependent scheduling theory we may consider the problems of repayment of multiple loans, fire fighting and maintenance assignments. In this book, we will discuss algorithms and complexity issues concerning various time-dependent scheduling problems.

Time-dependent scheduling is a relatively new subject. Although the first paper from the area appeared in late 1970s most results have been published in the last 10 years. So far, time-dependent scheduling has not gained much attention in books devoted to the scheduling theory. This book, summarizing the results of almost 15 years of the author's research into time-dependent scheduling, hopefully fills this gap.

The book is composed of 14 chapters, organized into four parts.

The first part of the book consists of five chapters and includes the mathematical background used in subsequent chapters. The aim of this part is to give the reader an introductory view of presented topics. Therefore, only

fundamental notions and concepts are discussed. In Chap. 1, the mathematical notation, the basic definitions and results used in this book are given. In Chap. 2, the essential concepts related to decision problems and algorithms are recalled. Chapter 3 includes the definitions and the most important results of the theory of  $\mathcal{NP}$ -completeness. This part is completed by Chaps. 4 and 5, where the basics of the scheduling theory and time-dependent scheduling theory are given, respectively. Each chapter of this part is completed with bibliographic notes including a list of selected references in which the reader may find a more comprehensive presentation of the particular topics.

The second part of the book includes a detailed survey of the time complexity of time-dependent scheduling problems. This part is composed of three chapters. In Chap. 6, single-machine time-dependent scheduling problems are discussed. Chapters 7 and 8 cover results concerning time-dependent scheduling on parallel and dedicated machines, respectively. Each chapter of this part is completed with the summary and tables.

The third part of the book is devoted to suboptimal algorithms for  $\mathcal{NP}$ -hard time-dependent scheduling problems. This part starts with Chap. 9, which presents approximation and heuristic algorithms. Chapter 10 introduces two greedy algorithms, which exploit the properties of the so-called signatures of sequences of job deterioration rates. Finally, local search heuristics for time-dependent scheduling problems are discussed in Chap. 11.

The fourth part of the book includes selected advanced topics in time-dependent scheduling. This part begins with Chap. 12, in which applications of matrix methods to time-dependent scheduling problems are discussed. Chapter 13 is devoted to scheduling proportionally and linearly deteriorating jobs under precedence constraints. In Chap. 14, closing the book, time-dependent scheduling problems with two criteria are studied.

Each chapter of these two parts ends with concluding remarks. Chapters of the fourth part include also comments on selected open problems.

The book is intended for researchers into the scheduling theory, Ph.D. students and everybody interested in recent advances in computer science.

The prerequisites for reading the book are the standard courses in discrete mathematics and calculus, fundamentals of the theory of algorithms and basic knowledge of any high-level programming language. Hence, this book can also be used by students of graduate studies.

The second part of the book can serve as a basis for an introductory course in time-dependent scheduling. The material from the next two parts can be used as a starting point for a research seminar in time-dependent scheduling.

The research presented in the book has been partially supported by grant N519 18889 33 of the Ministry of Science and Higher Education of Poland. While working on the book, I was also supported in different ways by different people. It is my pleasure to list here the names of the people to whom I am mostly indebted for help.

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Poznań, April 2008

*Stanisław Gawiejniewicz*

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