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BIOMEDICAL ENGINEERING**

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Topology in Molecular Biology

With 118 Figures, 6 in Color and 3 Tables

 Springer

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Foreword

The contents of this book focus on the recent investigations in molecular biology where applications of topology seem to be very stimulating. The volume is based on the talks and lectures given by participants of the three-month program “Topology in Condensed Matter”, which was held in the Max Planck Institut für Physik komplexer Systeme, Dresden, Germany, 8 May–31 July 2002, under the scientific direction of Professors M. Kléman, S. Novikov and myself. The aim of this program was to discuss recent applications of topology to several areas in condensed matter physics and molecular biology.

The first volume “Topology in Condensed Matter” is concerned with modern applications of geometrical and topological techniques to such new and classic fields of physics like electron theory of metals, theory of nano-crystals, aperiodic and liquid crystals, quantum computation and so on. This volume is published simultaneously in “Springer Series in Solid-State Physics”.

The present volume gives an exposition of the role of topology in the theory of proteins and DNA. The last thirty years affirmed very efficient applications of modern mathematics, especially topology, in physics. The union of mathematics and physics was very stimulating for both sides. On the other hand, the impact of mathematics in biology has been rather limited. However here also some interesting results were obtained. In particular, there are applications of knot theory in the theory of circular closed DNA. The recent discoveries in molecular biology indicate future successful applications of topology. For example, a reconstruction of three-dimensional protein structures by one-dimensional genomic sequences leads to very interesting and non-trivial combinatoric problems. There exist two “principa” reflecting the state of affairs in both fields: physics and biology in the recent past. The first one is the very popular concept of the famous physicist E. Wigner about “the unreasonable effectiveness of applications of mathematics in natural sciences (i.e. physics)”. Otherwise there exists the opposite opinion of the renowned contemporary mathematician I. Gelfand, who worked for many years in mathematical biology. He expressed the “unreasonable non-effectiveness of applications of mathematics in biology”. It is not to say that there are no applications

of mathematics to biology, but in Gelfand's view, no in-depth applications. May be the future development of both disciplines will disprove this joke. One indirect proof of this tendency is the contribution of Gelfand himself in this volume. Beside the biological chapter we add a mini-course of topology for physicists and biologists. We hope that this mathematical supplement makes this book self-contained and comprehensible for a more broad audience, including graduate and undergraduate students. Our biology chapter contains accounts of the recent interactions of topology and molecular biology – interactions with indeed much depth.

By the common opinion of participants the seminar was very successful. The organizers and participants are grateful to the MPIPKS for the generous sponsorship of the seminar with so unusual spectra of interest. Special thanks go to the directors of, MPIPKS Professors P. Fulde, J.-M. Rost and F. Julicher, the head of visitors' program Dr. S. Flach, the secretaries K. Lantch, M. Lochar and C. Poenish. We acknowledge our gratitude to the entire staff of the Institute for their help in organizing the seminar and for making sure it ran smoothly. We acknowledge our gratitude to Dr. C. Ascheron, who suggested publishing these lectures in Springer Verlag, and Sabrina Gauthamee Khan and K. Venkatasubramanian of SPi, Pondicherry and Adelheid Duhm who assisted in preparation of these books. The editor especially thanks Dr. L. Alania for his assistance in preparing this volume. We hope such programs that converge mathematicians, physicists, and biologists will continue.

Moscow-Dresden, November 2005.

Michael Monastyrsky

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