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Transcendental Methods in Algebraic Geometry

Lectures given at the 3rd Session of the
Centro Internazionale Matematico Estivo
(C.I.M.E.)

held in Cetraro, Italy, July 4–12, 1994

Editors: F. Catanese, C. Ciliberto



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Preface

The Third 1994 C.I.M.E. Session “Transcendental Methods in Algebraic Geometry” took place from July 4 to July 12 in the beautiful location of the Grand Hotel San Michele, Cetraro (Cosenza).

Already in the prehistory of algebraic geometry we find the theory of elliptic and Abelian integrals, which is directly linked with Riemann’s topological approach to creating the concept of a manifold. Later on, from Poincaré’s use of potential theory for the study of function theory on complex tori to Hodge’s theory of harmonic integrals and the vanishing theorems of Kodaira and others, we see that the transcendental approach puts many algebraic geometric questions on a firm basis.

In doing so, it establishes deep and surprising results, which can often be stated simply, invigorating a century-long tradition of manifold and fruitful relations with other disciplines. In a surprising way we see close analogies displayed between apparently distant methodologies, thus concretely augmenting the unified edifice of mathematics. It was one of the purposes of the 1994 course to look at the recent developments relating algebraic geometry to complex analysis, complex differential geometry, and differential topology as further manifestations of the core of algebraic geometry: a core which, although nourished by a myriad of subtle and intricate problems, has as its lifeblood the crucial interplay with a host of other subjects, be they physics, topology, algebra, analysis, differential geometry, or arithmetic.

From this point of view, the courses given by Demailly, Peternell, Tian, and Tyurin covered a very wide spectrum, each offering not only a broad view of recent developments and new results published here for the first time, but also opening wide perspectives still in the earliest stages of exploration. The beautiful texts of the four courses reproduced here give us ample justification for dispensing with further historical and mathematical description.

We would just like to recall that, as in the ancient Greek dramas, unity of place (lecturers and participants brought close together in the “golden cage” of San Michele), unity of action (there were only courses and problem sessions), and unity of time (one of the features of C.I.M.E.) contributed to the success of the course. This success was in large part due not only to the excellent lecturers but also to the brightness and knowledge of the participants: the variety of their cultural interests was for us very impressive, as well as their devotion to science amidst such tempting scenery.

The organizers: Fabrizio Catanese and Ciro Ciliberto

Table of Contents

Preface	v
Jean-Pierre DEMAILLY, L^2 Vanishing Theorems for Positive Line Bundles and Adjunction Theory	1
Thomas PETERNELL, Manifolds of Semi-positive Curvature	98
Gang TIAN, Kähler-Einstein Metrics on Algebraic Manifolds	143
Andrei TYURIN, Six Lectures on Four Manifolds	186