

Particle Accelerator Physics

Helmut Wiedemann

Particle Accelerator Physics

Third Edition

With 264 Figures

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To my sons and students

Preface

This issue of Particle Accelerator Physics is intended to combine the content of two earlier volumes and the volume on synchrotron radiation into one reference book. This book is designed for the serious scientist and student to acquire the underlying physics of electron accelerator physics. Introductory discussions on various types of accelerators have been eliminated, being well documented in the literature. Beam optics has been formulated in a general way as to be applicable also to proton and ion beams. Following the requests of many readers many solutions to exercises are given in the appendix. Breaking with the author's preference, Standard International units are used in this edition. In appendix 2, transformation rules are given to convert formulae between SI and cgs systems. In the process of rewriting the texts, known typographical and real errors have been corrected. The author wishes to express his sincere appreciation to all readers pointing out such errors.

I would like to thank all staff at Springer who have contributed to the publication of this text. Foremost, I thank Dr. Christian Caron for his suggestion and encouragement to combine several textbooks into one reference volume. For the expert editing and cover design I thank Mrs. Birgit Muench and her staff. Finally, it is a pleasure to thank Ms. Bhawna Narang from Techbooks for her patient and thorough preparation of the proofs and final printing.

Nakhon Ratchasima, Thailand
March 2007

Helmut Wiedemann

Preface to Volume I

The purpose of this book is to provide a comprehensive introduction into the physics of particle accelerators and particle beam dynamics. Particle accelerators have become important research tools in high energy physics as well as sources of incoherent and coherent radiation from the far infra red to hard x-rays for basic and applied research. During years of teaching accelerator physics it became clear that the single most annoying obstacle to get introduced into the field is the absence of a suitable textbook. Indeed most information about modern accelerator physics is contained in numerous internal notes from authors working mostly in high energy physics laboratories all over the world.

This text intends to provide a broad introduction and reference book into the field of accelerators for graduate students, engineers and scientists summarizing many ideas and findings expressed in such internal notes and elsewhere. In doing so theories are formulated in a general way to become applicable for any kind of charged particles. Writing such a text, however, poses the problem of correct referencing of original ideas. I have tried to find the earliest references among more or less accessible notes and publications and have listed those although the reader may have difficulty to obtain the original paper. In spite of great effort to be historically correct I apologize for possible omissions and misquotes. This situation made it necessary to rederive again some of such ideas rather than quote the results and refer the interested reader to the original publication. I hope this approach will not offend the original authors, but rather provides a broader distribution of their original ideas, which have become important to the field of accelerator physics.

This text is split into two volumes. The first volume is designed to be self contained and is aimed at newcomers into the field of accelerator physics, but also to those who work in related fields and desire some background on basic principles of accelerator physics. The first volume therefore gives an introductory survey of fundamental principles of particle acceleration followed by the theory of linear beam dynamics in the transverse as well as longitudinal

phase space including a detailed discussion of basic magnetic focusing units. Concepts of single and multi particle beam dynamics are introduced.

Synchrotron radiation, its properties and effect on beam dynamics and electron beam parameters is described in considerable detail followed by a discussion of beam instabilities on an introductory level, beam lifetime and basic lattice design concepts. The second volume is aimed specifically to those students, engineers and scientists who desire to immerse themselves deeper into the physics of particle accelerators. It introduces the reader to higher order beam dynamics, Hamiltonian particle dynamics, general perturbation theory, nonlinear beam optics, chromatic and geometric aberrations and resonance theory. The interaction of particle beams with rf fields of the accelerating system and beam loading effects are described in some detail relevant to accelerator physics. Following a detailed derivation of the theory of synchrotron radiation particle beam phenomena are discussed while utilizing the Vlasov and Fokker Planck equations leading to the discussion of beam parameters and their manipulation and collective beam instabilities. Finally design concepts and new developments of particle accelerators as synchrotron radiation sources or research tools in high energy physics are discussed in some detail.

This text grew out of a number of lecture notes for accelerator physics courses at Stanford University, the Synchrotron Radiation Research Laboratory in Taiwan, the University of Sao Paulo in Brazil, the International Center for Theoretical Physics in Trieste and the US Particle Accelerator School as well as from interaction with students attending those classes and my own graduate students.

During almost thirty years in this field, I had the opportunity to work with numerous individuals and accelerators in laboratories around the world. Having learned greatly from these interactions I like to take this opportunity to thank all those who interacted with me and have had the patience to explain their ideas, share their results or collaborate with me. The design and construction of new particle accelerators provides a specifically interesting period to develop and test theoretically new ideas, to work with engineers and designers, to see theoretical concepts become hardware and to participate in the excitement of commissioning and optimization. I have had a number of opportunities for such participation at the Deutsches Elektronen Synchrotron, DESY, in Hamburg, Germany and at the Stanford University at Stanford, California and am grateful to all colleagues who hosted and collaborated with me. I wished I could mention them individually and apologize for not doing so.

A special thanks goes to the operators of the electron storage rings SPEAR and PEP at the Stanford Linear Accelerator Center, specifically to T. Taylor, W. Graham, E. Guerra and M. Maddox, for their dedicated and able efforts to provide me during numerous shifts over many years with a working storage ring ready for machine physics experimentation.

I thank Mrs. Joanne Kwong, who typed the initial draft of this texts and introduced me into the intricacies of TEX typesetting. The partial support

by the Department of Energy through the Stanford Synchrotron Radiation Laboratory in preparing this text is gratefully acknowledged. Special thanks to Dr. C. Maldonado for painstakingly reading the manuscript. Last but not least I would like to thank my family for their patience in dealing with an "absent" husband and father.

Palo Alto, California
December 1992

Helmut Wiedemann

Preface to Volume II

This text is a continuation of the first volume on "Basic Principles and Linear Beam Dynamics". While the first volume has been written as an introductory overview into beam dynamics it does not include more detailed discussion of nonlinear and higher order beam dynamics or the full theory of synchrotron radiation from relativistic electron beams. Both issues are, however, of fundamental importance for the design of modern particle accelerators. In this volume beam dynamics is formulated within the realm of Hamiltonian dynamics leading to the description of multiparticle beam dynamics with the Vlasov equation and including statistical processes with the Fokker Planck equation. Higher order perturbations and aberrations are discussed in detail including Hamiltonian resonance theory and higher order beam dynamics. The discussion of linear beam dynamics in Vol. I is completed here with the derivation of the general equation of motion including kinematic terms and coupled motion. Building on the theory of longitudinal motion in Vol. I the interaction of a particle beam with the rf system including beam loading, higher order phase focusing and combination of acceleration and transverse focusing is discussed. The emission of synchrotron radiation greatly affects the beam quality of electron or positron beams and we therefore derive the detailed theory of synchrotron radiation including spatial and spectral distribution as well as properties of polarization. The results of this derivation is then applied to insertion devices like undulator and wiggler magnets. Beam stability in linear and circular accelerators is compromised by the interaction of the electrical charge in the beam with its environment leading to instabilities. Theoretical models of such instabilities are discussed and scaling laws for the onset and rise time of instabilities derived. Although this text builds up on Vol. I it relates to it only as a reference for basic issues of accelerator physics which could be obtained as well elsewhere. This volume is aimed specifically to those students, engineers and scientists who desire to acquire a deeper knowledge of particle beam dynamics in accelerators. To facilitate the use of this text as a reference many of the more important results are emphasized by a frame for quick detection. Consistent with Vol. I we use the cgs system of units.

However, for the convenience of the reader who is used to the system of international units conversion factors have been added whenever such conversion is necessary, e.g. whenever electrical or magnetic units are used. These conversion factors are enclosed in square brackets like $\sqrt{4\pi\epsilon_0}$ and should be ignored by those who use formulas in the cgs system. The conversion factors are easy to identify since they include only the constants $c, \pi, \epsilon_0, \mu_0$ and should therefore not mixed up with other factors in square brackets. For the convenience of the reader the source of these conversion factors are compiled in the appendix together with other useful tools.

I would like to thank Joanne Kwong, who typed the initial draft of this texts and introduced me into the intricacies of TEX typesetting and to my students who guided me by numerous inquisitive questions. Partial support by the Division of Basic Energy Sciences in the Department of Energy through the Stanford Synchrotron Radiation Laboratory in preparing this text is gratefully acknowledged. Special thanks to Dr. C. Maldonado for painstakingly reading the manuscript and to the editorial staff of Springer Verlag for the support during the preparation of this text.

Palo Alto, California
March 1994

Helmut Wiedemann

Preface to Synchrotron Radiation

This book covers the physical aspects of synchrotron radiation generation and is designed as a textbook and reference for graduate students, teachers and scientists utilizing synchrotron radiation. It is my hope that this text may help especially students and young researchers entering this exciting field to gain insight into the characteristics of synchrotron radiation.

Discovered in 1945, synchrotron radiation has become the source of photons from the infrared to hard x-rays for a large community of researchers in basic and applied sciences. This process was particularly supported by the development of electron accelerators for basic research in high energy physics. Specifically, the development of the storage ring and associated technologies resulted in the availability of high brightness photon beams far exceeding other sources.

In this text, the physics of synchrotron radiation for a variety of magnets is derived from first principles resulting in useful formulas for the practitioner. Since the characteristics and quality of synchrotron radiation are intimately connected with the accelerator and electron beam producing this radiation, a short overview of relevant accelerator physics is included.

In the first four chapters radiation phenomena in general and synchrotron radiation in particular are introduced based on more visual and basic physical concepts. Where exact formulas are required, we borrow results from rigorous derivations in Chaps. 9 and 10. This way the physics of synchrotron radiation can be discussed without extensive deviations into mathematical manipulations, which can be quite elaborate although straightforward. The consequence for the reader, of this dual approach to synchrotron radiation is that, here and there, one will find some repetitive discussions, which the author hopes will provide easier reading and continuity in the train of thought.

Chapters 5 to 8 give an overview of beam dynamics in storage rings and guidance for the optimization of a storage ring for synchrotron radiation production. The theory of synchrotron radiation is derived rigorously in Chap. 9 and that of undulator or insertion device radiation in Chap. 10. Finally, in Chap. 11 the physics of a free electron laser is discussed.

Each chapter includes a set of exercises. For those exercises which are marked with the argument (S), solutions are provided in Appendix A. In support of the practitioner utilizing synchrotron radiation most relevant formulas together with useful mathematical and physical formulae and constants are compiled in Appendices B–D.

The author would like to thank the editorial staff at Springer Verlag and especially Drs. H. Lotsch and C. Ascheron for suggesting the writing of this book. The trained eyes of Dr. A. Lahee and Ms. Dimler contributed much to minimize typographical errors and to greatly improve the overall appearance of the book. Special thanks go to Professors J. Dorfan and K. Hodgson at Stanford University for granting a sabbatical leave and to Professor T. Vilaithong at the Chiang Mai University in Thailand for providing a quiet and peaceful environment during the final stages of writing this book.

Chiang Mai, Thailand
December 2, 2001

Helmut Wiedemann

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