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INTRODUCTION

This volume contains lectures given at the Saint-Flour Summer School of Probability Theory during the period 7th - 23rd July, 1997.

We thank the authors for all the hard work they accomplished. Their lectures are a work of reference in their domain.

The School brought together 81 participants, 37 of whom gave a lecture concerning their research work.

At the end of this volume you will find the list of participants and their papers.

Finally, to facilitate research concerning previous schools we give here the number of the volume of "Lecture Notes" where they can be found :

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