Lecture Notes in Mathematics

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PREFACE

A symposium on Algebraic Groups took place at the University of Utrecht from 1 - 4 April, 1986. It was organized to celebrate two birthdays: the 350th anniversary of the university and the 60th birthday of its distinguished member, Professor T.A. Springer.

The university celebrated with a series of international scientific symposia and congresses, of which 'Algebraic Groups' was the first one. Our symposium was funded by the 'Stichting 350 jaar Rijksuniversiteit Utrecht', the 'Koninklijke Nederlandse Academie van Wetenschappen', and the Dutch research organisation ZWO. To the first of these bodies and to the convention bureau QLT employed by them, we are also indebted for help with the organization, as we are for secretarial help to the Mathematics Department of Utrecht University.

To honour Professor Springer, we felt it would be appropriate to invite a number of leading experts in the field of algebraic groups to lecture on their current research; in this way a wide spectrum of topics would be covered and the central rôle of algebraic groups in mathematics emphasized. It is a tribute to the active part which Springer has played in the development of the subject, that all of the speakers have had close scholarly and personal contacts with him at one time or another.

Of the fifteen invited speakers thirteen were able to come, while another mathematician graciously accepted to be a last-minute stand-in. Fourteen manuscripts were contributed to these Proceedings, which often, but not always, cover the subject of the talk delivered (cf. the list of talks and the table of contents below). They have been put in alphabetical order with respect to author's name(s) rather than in an order determined by subject. We briefly touch upon them here.

As the reader will notice, there are contributions on various topics centered around algebraic groups. Now that algebraic groups have been with us for about three decades, much is known about their structure (nevertheless, Tits contributes new information on unipotent subgroups of reductive groups in positive characteristic). Thus, attention has gone to subsequent questions, such as a structure theory for finite-dimensional algebraic groups. Popov's contribution shows that there is still little grip on a 'standard' example, such as Aut $\mathbb{A}_n$ (he constructs infinitely many nontriangular subgroups of $\mathbb{A}_n$ ($n \geq 3$) isomorphic to the 1-dimensional additive group $\mathbb{G}_a$). Nevertheless, many new insights have been obtained for special classes of infinite-dimensional groups such as Kac-Moody groups. The contribution by Kac & Peterson illustrates this. It also reflects the interest regained in invariant theory, a classical aspect of algebraic groups. In their paper, Le Bruyn & Procesi pay attention to this subject by studying the GL(n)-orbit space of the affine space of m-tuples of complex n by n matrices (on which the GL(n)-action is componentwise by conjugation). Richardson uses modern techniques of

invariant theory to derive an elementary necessary condition for normality of a closed subvariety of the Lie algebra of a semisimple group which is stable under the adjoint action of the group. Piatetski-Shapiro employs a Poincaré series for split reductive groups to produce Langlands L-functions. Geometric invariant theory is concerned with a description of the quotient space of a variety by a group acting on it. As a set on which this group acts, the variety can then be recovered from the quotient space and certain group data (the stabilizers). In the case where the group is an algebraic torus, Goresky & MacPherson indicate what kind of data suffice to reconstruct the variety as a topological space on which the group acts.

Invariant theory usually starts with a known group action on a variety. Representation theory on the other hand tries to describe all linear representations of a given group. The (finite-dimensional) representation theory of algebraic groups where the characteristic of the representation space coincides with that of the group is reasonably well understood, especially in characteristic 0. Part of the present interest in this area is directed to questions concerning special functions related to series of group representations. In his paper, Macdonald deals with a class of polynomial symmetric functions including the 'classical' Schur functions and zonal spherical functions related to various real forms of GL(n). In the cross characteristic representation theory, enormous progress has been made. In particular, for the finite subgroups of algebraic groups which are the fixed points of a (Frobenius type) automorphism, many class functions have been constructed which lead to characters. Lusztig, in his contribution, describes a Lie algebra variant of a special kind of class function he needed in his theory of character sheaves (in fact, functions vanishing outside the nilpotent variety, whose Fourier transforms have the same property). At the origin of the class functions in this theory are certain bundles on the flag variety. Borho exploits the theory of D-modules on the flag variety to study the orbits in the nilpotent variety and the classification of primitive ideals in the enveloping algebra of a semisimple Lie algebra. Relations with D-modules are also present in Brylinski's contribution, describing examples of cyclic homology of certain noncommutative algebras. The nilpotent variety, which turns up in so many contributions related to representation theory, also appears in Jantzen's survey of the determination of the cohomology of a restricted Lie algebra in positive characteristic.

There are also contributions on Lie groups, the ancestors of algebraic groups. Borel proves a theorem in unitary representations theory of Lie groups. It concerns the vanishing of relative Lie algebra cohomology, and such theorems are of importance for the cohomology of cocompact discrete subgroups. The latter groups plays a central role in the contribution of Mostow & Yau. They use Morse theory to compute homological invariants of the quotient of the unit ball - viewed as the positive cone in natural PU(1,2) space - by a discrete subgroup arising from the monodromy of multivariate hypergeometric functions.

As most of the lectures were excellent, they permitted the audience to gain a good impression of several areas of mathematics around the central theme of algebraic groups. The rather leisurely schedule of the symposium permitted many personal exchanges, and we hope the occasion stimulated and furthered the cause of research in the area.

We wish to thank the invited speakers for their talks and their manuscripts, and also all the other mathematicians who attended the symposium and helped to make it into a success. Contributors and editors alike take pleasure in offering this volume of Proceedings to Professor Springer as a token of esteem and friendship.

The Editors:

A.M. Cohen
W.H. Hesselink
W.L.J. van der Kallen
J.R. Strooker

List of Talks

Tuesday, April 1,

W. Borho, *Nilpotent orbits, primitive ideals and characteristic classes*

R.W. Richardson, *Invariant vector fields on a semisimple Lie algebra*

C. Procesi, *Matrices and invariant theory*

Wednesday, April 2,

J.L. Tits, *On rational unipotent elements of simple algebraic groups*

G.D. Mostow, *Some surfaces covered by the ball and a problem in finite groups*

G. Harder, *Cohomology and special values of L-functions*

I. Piatetski-Shapiro, *L-functions and automorphic forms on classical groups with Whittaker model*

Thursday, April 3,

G. Lusztig, *Fourier transforms on a semisimple Lie algebra over* $\mathbb{F}_q$

I.G. Macdonald, *Commuting differential operators and zonal functions*

J.C. Jantzen, *Restricted Lie algebra cohomology*

Friday, April 4,

J.-L. Brylinski, *Some examples of Hochschild and cyclic homology*

R.D. MacPherson, *The variety of complete quadrics*

V. Kac, *Unitary representations of Diff* S^1, *exceptional Lie algebras and statistical mechanics*

A. Borel, *A vanishing theorem in Lie algebra cohomology*

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