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Constrained
Global Optimization:
Algorithms and Applications

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Preface

Global optimization is concerned with the characterization and computation of global minima or maxima of nonlinear functions. The general constrained global minimization problem has the following form:

Given: $K \subseteq R^n$ compact set, $f:K \rightarrow R$ continuous function

Find: $x^ \in K$, $f^* = f(x^*)$ such that $f^* \leq f(x)$ for all $x \in K$*

Such problems are widespread in mathematical modeling of real world systems for a very broad range of applications. Such applications include economies of scale, fixed charges, allocation and location problems, quadratic assignment and a number of other combinatorial optimization problems. More recently it has been shown that certain aspects of VLSI chip design and database problems can be formulated as constrained global optimization problems with a quadratic objective function. Although standard nonlinear programming algorithms will usually obtain a local minimum to the problem, such a local minimum will only be global when certain conditions are satisfied (such as f and K being convex). In general several local minima may exist and the corresponding function values may differ substantially. The problem of designing algorithms that obtain global solutions is very difficult, since in general, there is no local criterion for deciding whether a local solution is global.

Active research during the past two decades has produced a variety of methods for finding constrained global solutions to nonlinear optimization problems. In this monograph we consider deterministic methods which include those concerned with enumerative techniques, cutting planes, branch and bound, bilinear programming, general approximate algorithms and large-scale approaches.

There has been a significant recent increase in research activity on the subject of constrained global optimization and related computational algorithms. This

monograph summarizes much of this recent work and contains an extensive list of references to papers on constrained global optimization, deterministic solution methods, and applications. We hope that this work will be valuable for other researchers in global optimization.

We wish to express our appreciation and thanks to Andrew T. Phillips and Nainan Kovoor who carefully read an earlier version of this monograph and suggested a number of valuable improvements. The authors' research described in this monograph was supported in part by the National Science Foundation Grant DCR8405489.

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