

# Lecture Notes in Control and Information Sciences

Edited by M. Thoma and A. Wyner

140

---

Z. Gajić, D. Petkovski,  
X. Shen

## Singularly Perturbed and Weakly Coupled Linear Control Systems

A Recursive Approach

---



Springer-Verlag  
Berlin Heidelberg New York  
London Paris Tokyo Hong Kong

**Series Editors**

M. Thoma · A. Wyner

**Advisory Board**

L. D. Davisson · A. G. J. MacFarlane · H. Kwakernaak  
J. L. Massey · Ya Z. Tsykin · A. J. Viterbi

**Authors**

Prof. Zoran Gajić  
Rutgers University  
Dept. of Electrical and Computer Engineering  
Piscataway, NJ 08855-0909  
USA

Prof. Djordjija Petkovski  
University of Novi Sad  
Faculty of Technical Sciences  
V. Vlahovića 3  
21000 Novi Sad  
YUGOSLAVIA

Dr. Xuemin Shen  
Rutgers University  
Dept. of Electrical and Computer Engineering  
Piscataway, NJ 08855-0909  
USA

ISBN 3-540-52333-2 Springer-Verlag Berlin Heidelberg New York  
ISBN 0-387-52333-2 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in other ways, and storage in data banks. Duplication of this publication or parts thereof is only permitted under the provisions of the German Copyright Law of September 9, 1965, in its current version, and a copyright fee must always be paid. Violations fall under the prosecution act of the German Copyright Law.

© Springer-Verlag Berlin, Heidelberg 1990  
Printed in Germany

The use of registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Printing: Mercedes-Druck, Berlin  
Binding: B. Helm, Berlin  
2161/3020-543210 Printed on acid-free paper.

---

---

## **PREFACE**

---

---

This book is designed to be a fairly comprehensive treatment of the recursive reduced-order methods for singularly perturbed and weakly coupled linear systems. There are numerous examples of singularly perturbed and weakly coupled dynamic systems that provide great challenges to engineers of different disciplines. Obvious examples of singularly perturbed and weakly coupled systems include electrical power systems, aerospace systems, large electric networks, process control systems in chemical and petroleum industries, etc. It is shown that the recursive reduced-order methods are applicable to wider classes of practical problems than the existing singularly perturbed and weakly coupled methods based on the power series expansion. The recursive methods offer several advantages. As it will be shown, the higher order of accuracy can be easily achieved at low cost, the parallel processing of information can be used, results are obtained under much milder assumptions (no analyticity requirements of the problem coefficients), the software and hardware implementation of the control algorithms is highly simplified due to complete parallelism in the design procedures.

This book is intended to the broad audience such as control engineers, applied mathematicians and advanced graduate students who seek a comprehensive view of the current developments in the theory of singularly perturbed and weakly coupled systems. The book emphasizes mathematical developments as well as their application to solving practical problems without assuming strong mathematical background of the readers.

To demonstrate the usefulness of the recursive reduced-order approach to the singularly perturbed and weakly coupled linear systems and to point out its various advantages we have included several real world examples: fluid catalytic cracker, twelve plate absorption column, magnetic type control system, F-8 aircraft, power system composed of two interconnected areas, distillation column, steam power system, and synchronous machine connected to an infinite bus.

#### IV

We hope that this book will help to reduce some of barriers that exist in recognizing the power and usefulness of the recursive reduced-order methods for singularly perturbed and weakly coupled linear systems, and it will help to broaden their implementation in practice.

Z. Gajić is indebted to his former advisors, Professors H. Khalil and J. Medanić, and to Professor P. Kokotović for bringing him into the challenging research areas of singular perturbations and weak coupling.

Grant support from the following sources is gratefully acknowledged for Dj. Petkovski from the U. S. - Yugoslav Joint Fund for Scientific and Technological Cooperation, in cooperation with the National Science Foundation Grant JF 736, and in cooperation with the Department of Energy under grant JF 727. Dj. Petkovski is particularly thankful to Professor M. Athans and to Dr. A. Levis for fruitful cooperation in the course of these two projects.

The authors are thankful for the contributions of T. Grodt, Professor V. Kecman, N. Harkara, W-C Su, and Đ. Tasevski.

Novi Sad, July 1989.

Authors

---

---

# TABLE OF CONTENTS

---

---

|   |    |
|---|----|
| <b>CHAPTER 1.</b>   |    |
| <b>INTRODUCTION</b>   | 1  |
| <b>CHAPTER 2.</b>   |    |
| <b>ALGEBRAIC LYAPUNOV AND RICCATI EQUATIONS</b>   | 7  |
| 2.1. Introduction   | 7  |
| 2.2. The recursive methods for singularly perturbed linear systems  | 8  |
| 2.2.1. The recursive reduced-order method for the algebraic Lyapunov equation   | 10 |
| 2.2.2. The recursive reduced-order method for the algebraic Riccati equation  | 13 |
| 2.3. The recursive methods for weakly coupled linear systems  | 18 |
| 2.3.1. The recursive reduced-order parallel algorithm for solving the algebraic Lyapunov equation of weakly coupled systems | 19 |
| 2.3.2. The recursive reduced-order parallel algorithm for solving the algebraic Riccati equation of weakly coupled systems  | 21 |
| 2.4. Decoupling transformation for weakly coupled linear systems  | 25 |
| 2.5. Conclusion   | 34 |
| <b>CHAPTER 3.</b>   |    |
| <b>OUTPUT FEEDBACK CONTROL OF LINEAR SINGULARLY PERTURBED AND WEAKLY COUPLED SYSTEMS</b>                                    | 35 |
| 3.1. Introduction   | 35 |
| 3.2. Output feedback for singularly perturbed linear systems  | 37 |
| 3.3. Case study: Fluid catalytic cracker  | 45 |
| 3.4. Output feedback for linear weakly coupled systems  | 47 |
| 3.5. Case study: Twelve plate absorption column   | 56 |

|  |     |
|--|-----|
| <b>CHAPTER 4.</b>  |     |
| <b>LINEAR STOCHASTIC SYSTEMS</b>   | 63  |
| 4.1. Recursive approach to singularly perturbed linear stochastic systems                            | 63  |
| 4.2. Case study: F-8 aircraft LOG controller   | 73  |
| 4.3. Recursive approach to weakly coupled linear stochastic systems                                  | 78  |
| 4.4. Case study: Electric power system example   | 89  |
| Appendix 4.1   | 92  |
| <b>CHAPTER 5.</b>  |     |
| <b>RECURSIVE APPROACH TO FINITE TIME SINGULARLY PERTURBED AND WEAKLY COUPLED CONTROL SYSTEMS</b>     | 98  |
| 5.1. Reduced-order recursive solution of the singularly perturbed differential Riccati equation.     | 98  |
| 5.2. Case study: The synchronous machine connected to an infinite bus                                | 109 |
| 5.3. Reduced-order recursive solution of the Riccati differential equation of weakly coupled systems | 111 |
| 5.4. Case study: The distillation column example   | 118 |
| Appendix 5.1   | 121 |
| <b>CHAPTER 6.</b>  |     |
| <b>APPLICATIONS TO THE DIFFERENTIAL GAMES</b>  | 125 |
| 6.1. Weakly coupled linear-quadratic Nash games  | 125 |
| 6.2. Solution of coupled algebraic Riccati equations   | 130 |
| 6.2.1. Zeroth-order approximation  | 131 |
| 6.2.2. Solution of higher-order accuracy   | 132 |
| 6.3. Numerical example   | 140 |
| Appendix 6.1   | 143 |
| <b>CHAPTER 7.</b>  |     |
| <b>LINEAR DISCRETE WEAKLY COUPLED CONTROL SYSTEMS</b>  | 144 |
| 7.1. Optimal reduced-order recursive solution of the weakly coupled discrete Riccati equation        | 144 |
| 7.1.1. Computational algorithm   | 145 |
| 7.1.2. Case study: Discrete model of the distillation column   | 147 |

## VII

|  |     |
|--|-----|
| 7.2. Recursive reduced-order solution of the stochastic linear weakly coupled discrete systems | 150 |
| 7.2.1. Linear-quadratic Gaussian control of discrete weakly coupled systems at steady state    | 151 |
| 7.2.2. Numerical example   | 160 |
| Appendix 7.1   | 161 |

### **CHAPTER 8.** **LINEAR DISCRETE SINGULARLY PERTURBED CONTROL SYSTEMS**

163

|   |     |
|---|-----|
| 8.1. Recursive solution of the discrete linear-quadratic control problem of singularly perturbed systems              | 163 |
| 8.1.1. Introduction   | 163 |
| 8.1.2. Reduced-order near-optimal solution of the discrete algebraic Riccati equation of singularly perturbed systems | 164 |
| 8.2. Near-optimal control of linear singularly perturbed discrete systems   | 171 |
| 8.2.1. Case study: Discrete model of F-8 aircraft   | 172 |
| 8.3. Parallel reduced-order controllers for stochastic linear singularly perturbed systems                            | 175 |
| 8.3.1. Introduction   | 175 |
| 8.3.2. Linear quadratic Gaussian control of discrete singularly perturbed systems at steady state                     | 176 |
| 8.3.3. Case study: Discrete steam power system  | 183 |
| 8.4. Conclusions  | 185 |
| Appendix 8.1  | 186 |
| Appendix 8.2  | 187 |

### **REFERENCES**

189