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150

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Robust Stabilization
in the Gap-topology

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Crescit scribendo scribendi studium

(Desiderius Erasmus)

PREFACE

'Crescit scribendo scribendi studium'. This experience I had, while writing this book. When I started, I thought I had everything clearly in mind, but, while writing, things turned out to be more complicated. The obligation to state results precisely, forced me to treat the subject in more detail than I planned. I hope however that a greater insight of the reader in the various notions and subtilty of the theory will be the gain of this effort. But it took me more paper to tell the story than I expected, and I hope that this final result is clarifying instead of tedious or boring to the reader.

Roughly speaking, this book can be divided into three main parts. In the first one, the notions of robust stabilization and the gap-topology are introduced. In the second part, most of the theory is developed. We find sufficient conditions for robust stabilization in the gap-topology, and are able to derive a method for the solution of the problem of optimally robust control. Finally, in the third part, algorithms are developed to actually compute a solution to this problem for systems in state-space form. So in this book we slowly tend from theory to practice.

Of course I'm grateful to all the people who helped me, in one way or the other, with the accomplishment of this paper. First of all I'd like to thank prof. M.L.J. Hautus. He awoke my interest for the field of systems theory and also attended me to this subject. I am also indebted to Siquan Zhu, who developed most of the theory, upon which my algorithms are based. Especially I would like to thank dr. Kees Praagman for carefully reading the manuscript and all his valuable suggestions. The cooperation with him was very stimulating and I'd like to thank him for always being available when I had questions or problems. I'm also grateful to drs. Anton Stoorvogel. When I didn't understand the computer anymore, he always got me out of trouble. Finally I'd like to mention drs. J. Timmermans. Without him, this book would probably have never been written. And last but not least, I thank my parents for all their support and encouragements. Without them I'd never come so far.

Luc Habets.

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NOTATION

Given a state-space realization $[A,B,C,D]$ of a plant P , the transfermatrix corresponding to P is given by

$$P(s) = D + C(sI-A)^{-1}B.$$

Throughout the text we use the notation $[A,B,C,D]$ not only to describe the realization of P , but also to indicate the related transfermatrix, i.e.

$$[A,B,C,D] = D + C(sI-A)^{-1}B.$$

Using this data-structure we can easily obtain the following formulae for operations on transfermatrices.

$$[A,B,C,D] = [T^{-1}AT, T^{-1}B, CT, D].$$

$$[A,B,C,D]^{-1} = [A-BD^{-1}C, BD^{-1}, -D^{-1}C, D^{-1}].$$

$$\begin{aligned} [A_1, B_1, C_1, D_1] + [A_2, B_2, C_2, D_2] &= \\ &= \left[\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, (C_1, C_2), D_1 + D_2 \right]. \end{aligned}$$

$$\begin{aligned} [A_1, B_1, C_1, D_1] * [A_2, B_2, C_2, D_2] &= \\ &= \left[\begin{bmatrix} A_1 & B_1 C_2 \\ 0 & A_2 \end{bmatrix}, \begin{bmatrix} B_1 D_2 \\ B_2 \end{bmatrix}, (C_1, D_1 C_2), D_1 D_2 \right] \\ &= \left[\begin{bmatrix} A_2 & 0 \\ B_1 C_2 & A_1 \end{bmatrix}, \begin{bmatrix} B_2 \\ B_1 D_2 \end{bmatrix}, (D_1 C_2, C_1), D_1 D_2 \right]. \end{aligned}$$