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Lie Methods in Optics II

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Lie Methods in Optics

THE SECOND WORKSHOP

COCOYOC, MÉXICO, JULY 18–22, 1988

After the first Lie Methods in Optics workshop,¹ informal contacts with the participants and readers of the proceedings volume² suggested the time was ripe for a second such gathering. In the intervening three and a half years, the applications of Lie algebras and groups to optics have spread and deepened. So it was recognized by Prof. E.C.G. Sudarshan, who agreed to co-chair the workshop. Unforeseen institutional problems prevented him from attending the event personally, but his line of work was well represented by Prof. R. Simon, from the Institute of Mathematical Sciences, Madras, India.

Lie methods have similarities and differences with the more traditional tools employed hitherto in light and magnetic optics; as a branch of mathematics, moreover, they are worthy of study by themselves. Chapter 1, written by PETER W. HAWKES, surveys the pre- and post-Lie landscapes of perturbation expansions. Information theory derives very succinctly from the Heisenberg algebra by metaplectic harmonic analysis. Chapter 2, by WALTER SCHEMPP, offers mathematical foundations for coherent optical computing in the form of parallel two-dimensional data compression, holographic image processing and interferometry, and neural architecture for pattern recognition.

Symbolic computation with Lie structures, extensively applied by many students and researchers originating from the University of Maryland, has yielded developments that enhance their use not only in magnetic and light optics, but in the very principles of perturbation expansions to high orders. The canonical integration and concatenation of Lie transformations are treated in Chapter 3 by ETIENNE FOREST and MARTIN BERZ, and Chapter 4 by ALEX J. DRAGT and LIAM M. HEALY. The Superconducting Super-Collider project has provided great impetus to the refinement of the theory and efficiency in the computation of aberration expansions of high rank in symplectic manifolds.

It is not lost upon us that optical computing is within the present technological ten-year horizon. Pending further developments in nonlinear and physical optics for gate circuitry —*photonics*—, the parallel and sequential

¹CIFMO-CIO Workshop on Lie Methods in Optics, held at León, México, January 7–10, 1985.

²*Lie Methods in Optics*, Proceedings, Ed. by J. Sánchez-Mondragón and K.B. Wolf. Lecture Notes in Physics, Vol. 250 (Springer, Berlin, Heidelberg, 1986).

processing capabilities of optical computers will require appropriate mathematics. The operations of convolution and correlation of two signals performed by purely optical means is the theme of Chapter 5, contributed by MARK KAUDERER out of his former association with MOSHE NAZARATHY and JOSEPH W. GOODMAN. The mathematics comes from paraxial optics but the symplectic algebra is applied to space and time.

A foundation of wide-angle optics based on the Euclidean group of motions is offered in Chapter 6, by K.B. WOLF. It accommodates geometrical and Helmholtz wave optics, defines a *wavization* process corresponding to—but distinct from—Heisenberg-Weyl quantization, and is compatible with signal Fourier analysis. (It is the privilege of the scientists in the less-developed countries to be able to roam almost free of technological necessities.) Its true aim is at present to contribute to the æsthetics, the coherence, and the extensions of Lie methods to untrodden fields with promise, particularly polarization tomography. Dr. VLADIMIR I. MAN'KO, one of the contributors to the first volume, who could unfortunately not be present at the second meeting, has again participated in a choice theme here.³ During February 1989, working together at IIMAS—CUERNAVACA, we were able to understand the relation between the Euclidean and Heisenberg-Weyl Lie optics. This is presented in Chapter 7. It was included in this volume because it establishes the bridge between the traditional turf of coherent state theory, paraxial optics including aberration expansions, and global 4π optics.

One topic that was severely missed in the first volume is the strand of polarization optics developed in India by N. Mukunda, R. Simon and E.C.G. Sudarshan. This time, unfortunately, it was personal tragedy that prevented their work from appearing here, and is the main factor in the delay in publication of these proceedings.

As in the first volume, the chapters were prepared by the workshop participants to be of lasting value to a wide audience, including mathematicians, group-theoretical physicists, as well as applied opticians. If Lie methods are to be used successfully, they should be not only understood by the prospective user, but provide him or her with the pleasure of revealing the hidden symmetries of Nature.

México, Summer 1989

K. B. Wolf

³In this case it was the local organizer's fault for changing the contemplated Winter meeting into a Summer one, according to correspondence with other participants, after the invitation was made—with due anticipation—through official channels.

Centro Internacional de Física y Matemáticas Aplicadas

In 1988 CIFMA initiated its international activities with a main program in Photonics, of which mathematical optics is a component. The Lie Methods in Optics Second Workshop took place in Cocoyoc, on July 19–22, 1988, with the informal discussion-room atmosphere of the first meeting (León, January 7–10, 1985). This volume collects the advances in the field.

CIFMA was created as a Civil Association in Cuernavaca, State of Morelos. It is intended that CIFMA develop in Mexico the manifold activities pioneered by the International Centre for Theoretical Physics in Trieste, Italy, with special attention to the perceived scientific and technological needs and strong points of this country. Within the Latin American region, the Cuernavaca center joins the network started decades ago by the Centro Latino Americano de Física (CLAF, Brazil) and Centro Internacional de Física (CIF, Colombia).

The persistent economic problems afflicting Mexico have crippled scientific institutions; national funds for scientific research have been particularly meager. For this reason we are most grateful to Dr. Salvador Malo, **Dirección General de Investigación Científica y Superación Académica** (1988), Ministry of Public Education, who also sponsored the first workshop, for the firm support of our endeavours.

The participation of Prof. R. Simon was possible through grants from the **Third World Academy of Sciences** (Trieste), and the **Fondo de Fomento Educativo BCH** (México DF) whom it is a pleasure to thank also for the visit of Dr. V.I. Man'ko, one of the contributors to this volume, co-beneficiary of the **Consejo Nacional de Ciencia y Tecnología**. The indispensable logistic support of Dr. Ignacio Méndez, **Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas**, and Dr. José Sarukhán, **Coordinación de la Investigación Científica** (1988, now Rector) of the **Universidad Nacional Autónoma de México** is gratefully acknowledged.

ABOUT THIS VOLUME: We may derive satisfaction from the state of scientific typography in Mexico. Isolated efforts of the past (*viz.* Lecture Notes in Physics volumes 189 [1983] and 250 [1986]) have yielded—at long last—a small but growing core of professional technical editors. The present volume was prepared in T_EX by **José Luis Olivares Vázquez** and **Arturo Sánchez y Gándara**, in the workshop of the **Sociedad Mexicana de Física**. The SMF publishes the *Revista Mexicana de Física* and the *Boletín de la SMF*, which we modestly point out were, in 1986, the first Latin American science journals to be composed with the efficiency and quality standards allowed by Donald Knuth's system. At the Third National Meeting of the *Grupo de Usuarios de T_EX* (Cuernavaca, January 1989) we counted over ninety volumes, including the *Enciclopedia de México*, being prepared or published. Since 1988, the President's Address to the Nation—together with all of its gory administrative annexes—are typeset in T_EX. The spearhead of these developments was **Aurión Tecnología**, whose Director, **Armando Jinich**, we thank for shoring up the funds for the present volume. For much good software, we thank **Max Díaz** and **Miguel Navarro Saad**.

ARTURO SÁNCHEZ Y GÁNDARA

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