

Lecture Notes in Mathematics

Edited by A. Dold, B. Eckmann and F. Takens

1406

L. Jacobsen (Ed.)

Analytic Theory of Continued Fractions III

Proceedings of a Seminar-Workshop,
held in Redstone, USA, June 26–July 5, 1988



Springer-Verlag

Berlin Heidelberg New York London Paris Tokyo Hong Kong

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Mathematics Subject Classification (1980): 01A55, 10A32, 10F35, 30B70,
33A30, 34A50, 40A15, 41A21

ISBN 3-540-51830-4 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-51830-4 Springer-Verlag New York Berlin Heidelberg

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Printed in Germany

Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.
2146/3140-543210 – Printed on acid-free paper

PREFACE

This volume contains the proceedings of a research Seminar–Workshop held in Redstone, Colorado from June 26 to July 5, 1988. The topic of this workshop was Analytic Theory of Continued Fractions, and it was organized by William B. Jones, University of Colorado and Arne Magnus, Colorado State University.

This was the third workshop of its kind. The first one was held in Loen, Norway in 1981 (proceedings published in LN in M N^o 932), the second one in Pitlochry and Aviemore, Scotland in 1985 (proceedings published in LN in M N^o 1199). The idea is that workers in the field shall come together to exchange ideas and start cooperation. So, in addition to the presentation of our latest results, we have talks on ideas, half–finished projects, ideas that did not work, etc. Questions and comments, stupid or not, are encouraged.

This time analysis of papers from the turn of the century was one of the issues. Mathematicians like Helge von Koch, Jan Sleszynski and Julius Worpitzky had results and arguments which still deserve attention. Indirectly, these studies led to the study of separate convergence, a topic presented in a survey article in this volume, and to a historical article on Julius Worpitzky. In addition we continued discussions on nearness problems, the connection to Padé Approximants, and applications to number theory and differential equations. Interesting in this respect is for instance the use of Lange's δ –fractions to solve Riccati equations, and the question of finding thin subsets E_0 of an element region E whose best limit region V_0 is dense in the best limit region V for E . Also several other topics which are not reflected in these proceedings were discussed and will hopefully be published elsewhere.

It is evident that for workshops like this the location is important. Comfort, "isolation", a good seminar room and soothing nature are essential. We had all this in beautiful Redstone, and we are grateful to Nancy Lambert, manager of the Redstone Inn, for her hospitality. We also wish to express our gratitude to the Norwegian Research Council for Science and the Humanities, to the U.S. National Science Foundation, to the University of Colorado and to the universities of the respective participants for financial support. Finally we would like to thank Professor B. Eckmann for accepting this volume for publication.

Lisa Jacobsen

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