

# Lecture Notes in Mathematics

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J.-P. Francoise R. Roussarie (Eds.)

## Bifurcations of Planar Vector Fields

Proceedings of a Meeting held in Luminy, France,  
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## PREFACE

The meeting held in Luminy in September 18-22, 1988 brought together most of the world's specialists in bifurcations of vector fields of the plane. The main subjects of the theory were discussed, including:

- Finiteness of the number of limit cycles of ordinary differential equations in the plane. The problem of Dulac is that of determining whether polynomial vector fields have a finite number of limit cycles. One solution is presented in this volume in the framework of new and much farther-reaching methods for the study of differential equations, such as accelero-summation.
- Multiplicity of polycycles. Their definition seems to be a first step towards the solution of Hilbert's 16th problem produced to prove the existence of a uniform bound, dependent only on degree, for the number of limit cycles.
- Zeroes of abelian integrals. This is a topic which links up directly to real algebraic geometry. It intervenes in an infinitesimal version of Hilbert's 16th problem, and also in the question of enumeration of critical points of the period for which is important in the study of bifurcations.
- Numerical simulation and symbolic computation on computer in the study of differential equations.
- The work (in particular of Chinese groups of researchers) on quadratic equations, that pick up again classical methods of bifurcation theory such as the method of rotations.
- Modelling of predator-prey ecological systems. The subject is in widespread use in biomathematics to describe biological cycles.
- The use of methods of non-standard analysis in the study of bifurcation with delay.

The articles in this volume will initiate the reader quickly to the most recent result in this field at the interface of fundamental mathematics and of its applications, currently in full development.

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