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H. J. Zwart

Geometric Theory for Infinite
Dimensional Systems



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Geometry may sometimes appear to take the lead over analysis but in fact precedes it only as a servant goes before the master to clear the path and light him on his way.

James Joseph Sylvester

PREFACE AND ACKNOWLEDGEMENT

In the spring of 1984 I started with my research on geometric theory for infinite dimensional systems. The research topic was suggested to me by Ruth Curtain, who had done some preliminary investigations on this topic. Many questions were at that time still open and a more fundamental theory was still missing. We knew that the key-concept in geometric theory for finite dimensional systems, that is (A,B) -invariance, has lost its strength for infinite dimensional systems. So I began to look for different concepts which would be more appropriate for infinite dimensional systems. It turned out that these were the concepts of open-loop invariance and frequency invariance. Although the concept of frequency invariance had already been introduced for finite dimensional systems by Hautus, he did not give it any special name. I have chosen this name, since this expresses in a concise way that this is an invariance concept in the frequency domain. Once the equivalence between open-loop, frequency, and closed loop invariance was established, the solvability of various disturbance decoupling problems came relatively easy. In this monograph three disturbance decoupling problems are studied: the Disturbance Decoupling Problem (DDP), the Disturbance Decoupling Problem with Measurement Feedback (DDPM) and the Disturbance Decoupling Problem with Measurement Feedback and Stability (DDPMS). The theory can easily be extended to other disturbance decoupling problems, with the notable exception of the 'almost' version, which are studied in the finite dimensional case by Willems and Trentelman, see e.g. [39]. The theory for the almost disturbance decoupling problems is one of the main still open problems in geometric theory for infinite dimensional systems.

The monograph is addressed to researchers in the field of geometric theory of infinite dimensional systems. In this book I shall use basic concepts of the infinite dimensional system theory as C_0 -semigroup, approximate controllability, initial observability, which are covered in the second and third chapter of Curtain and Pritchard [9]. This book is self-contained with respect to the notions of the geometric theory, although sometimes we shall refer to the references for the finite dimensional case.

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Although it may seem that writing a monograph and doing research is a solo occupation, in reality it is a team occupation and I owe my team members of the Groningen System Theory Group a great debt of gratitude.

First of all I want to thank Ruth Curtain who found always the time and the patience to listen to my ideas. Her guidance made sure that my research would not wander off in queer directions.

During the past four years it has been a great pleasure to share the office with Jan Bontsema. As a room-mate he always had a lending ear to listen to my (sometimes obscure) problems and his relativizing way of looking at these problems really meant a lot to me. Furthermore I would like to thank the other members of the System Theory Group in Groningen; Harry Croon, Christiaan Hey, Hans Nieuwenhuis, Paula Rocha, Siep Weiland and Jan Willems, for the privilege of working with them. They all contributed in their own way to this research and made our lunch breaks a very cosy hour. I also express my gratitude to Hans Schumacher whose insight into the problem plus his remarks and ideas helped me to get my research started.

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*Groningen,
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Hans Zwart

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