

Foundations of Engineering Mechanics

**V. A. Svetlitsky, Statistical Dynamics and Reliability Theory for
Mechanical Structures**

Springer

Berlin

Heidelberg

New York

Barcelona

Hongkong

London

Mailand

Paris

Tokio

Engineering  **ONLINE LIBRARY**

<http://www.springer.de/engine-de/>

V. A. Svetlitsky

Statistical Dynamics and Reliability Theory for Mechanical Structures

Translated by N. L. Reshetov

With 189 Figures



Springer

Series Editors:

Vladimir I. Babitsky
Loughborough University
Department of Mechanical Engineering
LE11 3TU Loughborough Leicestershire
United Kingdom

J. Wittenburg
Universität Karlsruhe (TH)
Institut für Techn. Mechanik
Kaiserstr. 12
76128 Karlsruhe
Germany

Author:

V. A. Svetlitsky
Bauman Moscow State Technical University
The Department of Applied Mechanics
2-nd Baumanskaya St. 5
107005 Moscow
Russia
e-mail: voronov@rk5.bmstu.ru

Translator:

Nikolay Reshetov
25-1-56, 13-th Parkovaya St.
105215 Moscow
Russia

ISBN 978-3-642-53657-1 ISBN 978-3-540-45826-5 (eBook)
DOI 10.1007/978-3-540-45826-5

Cataloging-in-Publication Data applied for

Bibliographic information published by Die Deutsche Bibliothek
Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed
bibliographic data is available in the Internet at <<http://dnb.ddb.de>>.

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in other ways, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution act under German Copyright Law.

Springer-Verlag Berlin Heidelberg New York
a member of BertelsmannSpringer Science + Business Media GmbH

<http://www.springer.de>

© Springer-Verlag Berlin Heidelberg 2003
Softcover reprint of the hardcover 1st edition 2003

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: Camera ready by author
Cover-Design: de'blik, Berlin
Printed on acid free paper 62/3020/kk - 5 4 3 2 1 0

Preface

The monograph text is based on lectures delivered by author during many years for students of Applied Mechanics Department of Bauman Moscow State Technical University. The monograph includes also analytical results of scientific research obtained in collaboration with industry.

Progress in developing new equipment has called for a better understanding of the physical peculiarities pertaining to the action of designed structures in real conditions. This is necessary for increasing the accuracy of the analysis and making these structures more reliable.

It has been found that classical determined perturbations are not principal and that determinism-based methods of classical mechanics prove insufficient for understanding and explaining physical effects that arise at the operation of instruments located on moving objects, the vibration of rocket engines, the motion of a vehicle, and the action of wind and seismic loads. Therefore the necessity arose for devising a new physical model to analyze these dynamic processes and, in particular, for creating a new mathematical apparatus that would allow us to take into account non-deterministic external excitations. The theory of random processes that had been developed well enough as applied to problems of radio engineering and automatic control, where the effect produced by random excitations appeared to be commensurable with that of deterministic excitations and where the ignoring of the random excitations would bring about incorrect results, became such an apparatus. Therefore the theory of random processes began to be used for the solution of specific problems relevant to radio engineering, in particular, earlier than for the analysis of mechanical systems where the random excitations were frequently being ignored. In many applied problems such ignoring of random excitations, the more so really small ones, is quite allowable and their solution does not require the use of statistical mechanics. If, however, random excitations are comparable (in probability characteristics) with known forces and especially when only random excitations act on the system, the classical methods of analysis become unacceptable and the obtainment of numerical results demands the use probability-based methods. Therefore introducing statistical methods of analysis in practical design is a necessary condition of making reliable structures.

It is impossible to increase the reliability of new equipment designed for operation in extreme conditions and meeting the most stringent requirements to operation accuracy and reliability, and at the same time to achieve a reduction in the material input per unit of consumption, without the use of statistical mechanics.

The first four chapters are devoted to the fundamentals of the theory of probabilities and random processes. Consideration is given to random quantities and functions and their probability characteristics; probability density distribution functions; and mathematical expectations and variances. Different types of distribution laws encountered in practical problems are expounded. Non-stationary and stationary random processes that are of practical importance in analyzing the vibrations of mechanical systems are examined. Principal results of the special theory of stationary random functions and the implementation of spectral presentation of stationary random functions at steady-state vibration analysis are described, and the theory of Markovian processes is discussed.

Chapters 5–8 are concerned with the random vibrations of systems with a finite number of degrees of freedom and of systems with distributed parameters. The theory of random vibrations is presented similarly to the classical theory of vibrations that allows us to establish in the most obvious manner in what way these sections of mechanics (determined vibrations and random vibrations) are related and where they differ. The methods of analyzing random vibrations presented in the textbook make it possible to investigate dynamic processes that arise in mechanical systems (i.e. to determine the probabilistic characteristics of the generalized coordinates of a system and of their derivatives for a system with a finite number of degrees of freedom, and to obtain the probabilistic characteristics of the state of stress and strain of a system with distributed parameters).

The ninth chapter deals with the fundamentals of the theory of reliability and analyzes numerical methods of determining the probability of no-failure operation of mechanical systems under a single or small number of loading cycles and at a confined time of the process.

The use of methods of statistical mechanics in practical design becomes a possibility only when statistical information about random perturbations is available, but there are cases where we cannot obtain such information to the extent required for the design. In our presentation of the theoretical fundamentals of statistical mechanics we assume that the necessary information about random perturbation is known. In engineering practice, however, the situation may be different due to a very large volume of experimental investigations involved that sometimes turns out to be unfeasible because of technical difficulties or inadequate financing. The problem of obtaining the probability characteristics of perturbations is much more difficult than that of finding a subsequent solution to equations of the state of a system. Therefore this textbook contains a chapter expounding the theory and numerical

methods of analyzing problems of the dynamics of mechanical systems when the available information about random perturbations is insufficient for a design that uses statistical mechanics.

Moscow, June 2002

Valery Svetlitsky

Contents

Introduction	1
1. Fundamentals of the Probability Theory and the Theory of Random Processes	11
1.1 Brief Information on the Probability Theory	11
1.1.1 Basic Concepts of the Probability Theory	11
1.2 The Distribution Function and the Probability Density of a Random Variable	14
1.3 Numerical Characteristics of Random Quantities and Their Principal Properties	18
1.4 Probability Density Distribution Laws	22
1.5 Determination of the Probability of a Normally Distributed Random Quantity Lying in the Given Range	30
1.7 Complex Random Quantities	40
1.8 Numerical Characteristics of Functions of Random Arguments	42
2. Non-Stationary Random Functions (Processes)	45
2.1 Introduction	45
2.2 Probability Characteristics of Non-Stationary Random Functions	48
2.3 Random Function Systems and Their Probability Characteristics	55
2.4 Random Functions Linear Transformations	59
2.5 The Probabilistic Characteristics of the Linear Differential Equations at Non-Stationary Random Disturbances	65
3. Stationary Random Functions (Processes)	73
3.1 Probability Characteristics of Stationary Random Functions ..	73
3.2 The Ergodic Property of a Stationary Random Function	77
3.3 Derivatives and Integrals of Stationary Functions	81
3.3.1 Probability Characteristics of Stationary Random Function Derivatives	81
3.3.2 Probability Characteristics of the Integral of Stationary Random Functions	83

- 3.4 The Spectral Representation of Stationary Random Processes 84
 - 3.4.1 Spectral Densities of Stationary Function Derivatives . 88
 - 3.4.2 Determination of Spectral Density (Examples) 88
- 3.5 Cross-Spectral Densities and their Properties 92
- 3.6 Determination of the Spectral Densities of the Linear Differential Equations with Constant Coefficients Solutions 95

- 4. Fundamentals of the Markov Processes Theory 101**
 - 4.1 Continuous One-Dimensional Markov Processes 101
 - 4.2 The Fokker–Planck–Kolmogorov Equation 103
 - 4.3 Multidimensional Markov Processes 119
 - 4.4 Determination of the Probability of Attaining a Random Function Possible Values Area Boundaries 123

- 5. Random Vibrations of Systems with One Degree of Freedom 131**
 - 5.1 Free Random Vibrations of Linear Systems 131
 - 5.2 Forced Random Vibrations of Linear Systems 136
 - 5.2.1 Non-Stationary Vibrations 136
 - 5.2.2 Stationary Forced Vibrations 152
 - 5.3 Vibrations Caused by Random Kinematic Excitation 158
 - 5.3.1 Non-Stationary Random Vibrations at Kinematic Excitation 159
 - 5.4 The Problem of Overshoots at Random Vibrations 171
 - 5.5 Nonlinear Random Vibrations 182
 - 5.5.1 The Method of Statistical Linearization 183
 - 5.5.2 The Solution of the Nonlinear Equations with the Use of Markov Processes 189
 - 5.5.3 The Method of Statistical Trials (Monte-Carlo Method) 192

- 6. Random Vibrations of Systems with Finite Number of Degrees of Freedom 197**
 - 6.1 Free Random Vibrations of Linear Systems 197
 - 6.2 Vibrations at Random Pulse Loading 202
 - 6.3 Non-Stationary Random Vibrations of Linear Systems 217
 - 6.4 The Method of Principal Coordinates in Non-Stationary Vibrations Analysis 222
 - 6.5 Forced Stationary Random Vibrations of Linear Systems 240

- 7. Random Vibrations of Strings; Longitudinal and Torsional Vibrations of Straight Rods 259**
 - 7.1 Introduction 259
 - 7.2 Equations of Small Vibrations 263
 - 7.3 Solving Equations of Small Vibrations 267

8. Random Vibrations of Rods	279
8.1 Nonlinear Equations of Motion of Three-Dimensional Curvilinear Rods	281
8.2 Equations of the Motion of a Rod in the Attached Coordinate System	286
8.2.1 Equation of Space Motion of a Rod	286
8.2.2 Equation of Plane Motion of a Rod	288
8.2.3 Rods Having Lumped Masses	289
8.3 Equation of Small Vibrations of Rods	290
8.3.1 Equations of Small Vibrations in the Attached Coordinate Frame	293
8.3.2 Equations of Small Vibrations about a Natural State	294
8.4 Determination of Eigenvalues and Eigenvectors	297
8.5 Non-Stationary Random Vibrations of Rods	299
8.6 Stationary Random Vibrations of Rods	303
9. Fundamentals of Reliability Theory	313
9.1 Introduction	313
9.2 Elementary Problems of Reliability Theory	322
9.3 Possible Causes of Failures	324
9.4 Determination of Numerical Values of No-Failure Operation Probability (Reliability)	325
9.5 Determination of Reliability at the Linear Dependence of a Stress State on Random Loads	337
9.6 Determination of the Probability of No-Failure Operation at the Nonlinear Dependence of the Random Quantity F on External Loads	339
10. Random Processes at the Action of Random Functions Bounded in Absolute Value	349
10.1 Introduction	349
10.2 Determining the Maximum Values of the Components of the Systems State Vector	359
10.3 Areas of Possible Values of the System State Vector at the Action of Independent Excitations	361
10.4 Projections of the Area of Possible Values of the System State Vector onto Two-Dimensional Planes	372
10.5 Determination of the Maximum Values of Dynamic Reactions	374
10.6 Areas of Possible Values of the System State Vector in the Case of Several Sections of Motion	380
10.7 Areas of Possible Values of the System State Vector at the Action of Dependent Random Excitations	384
10.8 Determination of the Maximum Values of Linear Functionals at Independent Excitations	403

XII Table of Contents

10.9 Maximum Value of a Linear Functional at Dependent Excitations	412
10.10 Vibration Protection of Mechanical Systems	418
A. Appendices	431
A.1 Elementary Generalized Functions	431
A.2 Values of Integrals J_n	434
A.3 Correlation Functions and Spectral Densities Corresponding to Them	436
A.4 Hiawatha Designs an Experiment	438
References	443
Index	445