Lecture Notes in Mathematics

Edited by J.-M. Morel, F. Takens and B. Teissier

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- an informative introduction, with adequate motivation and perhaps some historical remarks: it should be accessible to a reader not intimately familiar with the topic treated;
- a subject index: as a rule this is genuinely helpful for the reader.

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