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A.G. Gorshkov, D.V. Tarlakovsky

Transient Aerohydroelasticity of Spherical Bodies

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Translated by E. Evseev and V. Balmont

With 65 Figures



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Series Editors:

V. I. Babitsky
Department of Mechanical Engineering
Loughborough University
LE11 3TU Loughborough, Leicestershire
United Kingdom

J. Wittenburg
Institut für Technische Mechanik
Universität Karlsruhe (TH)
Kaiserstraße 12
76128 Karlsruhe / Germany

Authors:

Anatoly G. Gorshkov
Moscow Aviation Institute
Dept. of Applied Mechanics
Volokolamskoe Shosse 4
125871 Moscow, Russia
e-mail: bugaev@mai.ru

Dmitry V. Tarlakovsky
Moscow Aviation Institute
Dept. of Applied Mechanics
Volokolamskoe Shosse 4
125871 Moscow, Russia
e-mail: bugaev@mai.ru

Translators:

Evgeny G. Evseev
Shodnenskaya Street, 50-34
123363 Moscow, Russia

Vladimir V. Balmont
Kosmodemiyanskikh Street, 6-125
125171 Moscow, Russia

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Preface

The problems of transient interaction of deformable bodies with surrounding media are of great practical and theoretical importance. When solving the problems of this kind, the main difficulty is in the necessity to integrate jointly the system of equations which describe motion of the body and the system of equations which describe motion of the medium under the boundary conditions predetermined at the unknown (movable) curvilinear interfaces. At that, the position of these interfaces should be determined as part of the solution process. That is why, the known exact solutions in this area of mechanics of continuum have been derived mainly for the cases of idealized rigid bodies.

Different aspects of the problems of transient interaction of bodies and structures with continuum (derivation of the efficient mathematical models for the phenomenon, development of the theoretical and experimental methods to be used for study of the transient problems of mechanics, etc.) were considered in the books by S.U. Galiev, A.N. Guz, V.D. Kubenko, V.B. Poruchikov, L.I. Slepyan, A.S. Volmir, and Yu.S. Yakovlev. The results presented by these authors make interest when solving a great variety of problems and show a necessity of joint usage of the results obtained in different areas: aerohydrodynamics, theory of elasticity and plasticity, mechanics of soils, theory of shells and plates, applied and computational mathematics, etc.

We start this book with a presentation of the main dependences and relationships of the theory of transient interaction of elastic bodies with continuum (Chap. 1).

Throughout, we consider only the bodies and compound structures having spherical surfaces. For many cases of their interaction with elastic and acoustic media, we derive exact analytical solutions (Chapters 2–4 and 6). These solutions, both in the cases of external and internal problems, are derived in the form of series expansion in terms of the Legendre polynomials. We propose an algorithm for determination of the coefficients of series expansion of the displacements and stresses into Legendre polynomials in a form of superposition of *generalized spherical waves*, and this is a generalization of the known method of summation of elementary spherical and plane waves used for solving one-dimensional problems (Chapters 3 and 5). At that, the

governing equations for the problems under consideration are the recursive relationships in the space of the Laplace transform with respect to time for arbitrary functions corresponding to converging and diverging waves. Contrary to the traditional approximate computation of the integrals of convolution type, we present here an exact algorithm of inversion of the Laplace transform for the class of the problems under study (Sects. 2.2 and 3.5).

We derive the new representations for the transition functions of diffraction of acoustic waves by spherical obstacles in a form of the solution of an ordinary differential equation derived using the integral for generalized spherical waves, and in a form of the improper integral derived using the spectrum theory of self-adjoint differential operators (Sect. 4.1).

Based on the solution of the general problem, we obtain the solutions for various limiting cases of great practical importance (Chapters 2, 5, and 6).

Because of the complexity of derivation of analytical solutions of the problems on penetration of deformable structures into fluid (the complexity is mainly caused by significant change in shape of contacting and free surfaces, appearance and propagation of the zones of cavitation in the fluid, and elastic-plastic deformations in the structure's material), we regard the numerical methods, particularly, the methods of finite differences and finite elements, to be the most promising.

In Chap. 7, we consider one of the possible variants of numerical solution of the problem on penetration of elastic spherical shell into a half-space occupied by an ideal compressible fluid. At that, we use the finite difference scheme of the second order of approximation with respect to time and the spatial coordinates.

The authors are very thankful to N.I. Drobyshevsky, who performed numerical calculations, the results of which are presented in Chap. 7.

The monograph is addressed to scientists, institutional and industrial researchers, lecturers, and graduate students.

Moscow, October 2000

Anatoly Gorshkov
Dmitry Tarlakovsky

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