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Foundations of Engineering Mechanics

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Foreword

Books like their authors have their destinies. This book for the most part could have been written earlier, but it happened to be released as late as the 21st century. It is based on the numerous fundamental studies of my teacher Kamil Sh. Khodzhaev. His disease prevented him from implementing his ideas in the form that he had been contemplating for years. It was me who tried to convey his concepts and ideas with the least possible distortions.

This book contains a number of solutions worked out by Kamil Sh. Khodzhaev himself as well as problems solved by the authors jointly or separately.

Kamil Sh. Khodzhaev founded St. Petersburg school of electromechanics, with the focus on mechanics as a part, however distinctive, of general analytical mechanics, having been inspired by the desire of our common Teacher Anatoly I. Lurie “to set order” in this significant branch of science.

Khodzhaev’s school has many followers and their work is an integral part of the book. Some original ideas and studies of our colleagues are provided with footnotes in the corresponding sections while Chapter 6 dealing with the motion of the charged particle in electromagnetic field was written in cooperation with another co-author Alexander G. Chirkov.

I would like to express my deep gratitude to the colleagues and students, without whose contribution many problems analyzed in this book would have been left unsolved. My deep gratitude goes to our friends and colleagues Iliya I. Blekhman, Robert F. Nagaev, Dmitry A. Indeitsev as well as the staff of the Department of Mechanics and Control Processes of State Polytechnical University of St. Petersburg for their helpful suggestions,

understanding and unfailing support rendered in the process of working at this book. Sincere gratitude is expressed to Alexander K. Belyaev who not only took the trouble of translation into English and preparing the camera-ready manuscript, but who also made a number of important suggestions for its improvement.

Dmitry Yu. Skubov

Introduction

The systems with essentially interacting mechanical and electromagnetic processes are referred to as the electromechanical systems. There is a great amount of applications of such systems in modern technology. It is worth mentioning the converters of mechanical energy into electric energy (various types of generators, electromagnetic vibration exciters, induction sensors etc.) and the inverters of electric energy into mechanical one (electric motors, catapults, electric-type instrument etc.). For description of processes in the electromechanical system one needs prescribing the generalized coordinates and velocities of the mechanical system and the electromagnetic field created by magnets or currents at any time instant. The sources of the magnetic field can be a part of the mechanical system and move together with it. An alternative is that they do not appear in the mechanical system and in this case their motion and position are described independently. For example, in the electric motor or generator a part of conductors (the windings) moves together with the rotor whereas another part is located on the motionless stator.

Electromechanical systems can be split into two classes: the systems with (i) distributed and (ii) concentrated currents. The systems of the first type describe motions of a conducting rigid body in the magnetic field (magnetic suspensions, bearings, systems for orientation) whereas the second type of electromechanical systems deals with the systems with linear currents (conductors). There are various approaches to investigation of dynamics of the electromechanical systems and this difference is especially evident under consideration of the systems with distributed currents. The first approach is connected with solving of the coupled electrodynamical (Maxwell

equations) and mechanical problems, an extensive bibliography being resulted in [80]. Another approach is based on the discrete description of the electromagnetic field by representing the distributed whirling currents in the form of a finite-dimensional or infinite-dimensional systems of conducting contours [90]. For such a description the vector of magnetic induction and the vector of intensity of the electric field are expressed in terms of a finite or countable set of other quantities. They are the effective charges and contour currents which are analogous to the generalized coordinates and generalised velocities. In the framework of this approach, the conditions of quasi-stationarity ignoring the electromagnetic waves generated by motion of the charges should be satisfied. The second approach is more natural for the case in which an electric subsystem consists of a set of linear conductors in the electric circuit.

Extensive literature is devoted to dynamics of the electromechanical systems. Among the literature in Russian it is necessary to mention, first of all, a chapter “Dynamics of nonholonomic systems and general theory of electric machines” in the classical book “Dynamics of nonholonomic systems” by Yu.I.Neimark and N.A.Fufaev. This work laid the groundwork for the fundamental principle of the investigation of the electromechanics considered nowadays as a part of general mechanics and analytical mechanics. An important role in the development of electromechanics played the book by Yu.G.Martynenko “Motion of a rigid body in electric and magnetic fields”. One of the principal results of this book was development of the method of splitting the coupled problem of calculation of the field and motion of a suspended rigid body in two extreme cases: (i) high-frequency and (ii) quasi-static magnetic field. The book “Electromechanical systems” by A.Yu.Lvovich published in 1989 is more likely a textbook.

Among the books of the foreign authors it is worth noting the book “Electromechanical systems” by A.Lenk and “Electromechanical energy converters” by D.C.White and H.H.Woodson.

The works of the Nizhni Novgorod school of mechanics, namely the works by G.G.Denisov, N.V.Derendyaev, O.D.Pozdeev, Yu.M.Urman and others played a significant role in development of the theory of electromechanical systems and especially the theory of electromagnetic suspension. It is also necessary to mention a series of works by V.V.Beletsky on the theory of motion of the magnetised satellite and the works by A.A.Burov, V.V.Kozlov and V.A.Samsonov on the general problem of motion of a rigid body in the magnetic field.

Extensive literature of the mechanical-mathematical character is devoted to the theory of electric machines. Here, first of all, it is necessary to notice the results by G.A.Leonov and the groups of scientists of Petersburg State University on the qualitative theory of equations of the electric machines and the works of scientists of the Illinois Center of Electromechanics (P.V.Kokotovic, P.W.Sauer, M.A.Pai et al) on investigation of dynamics of synchronous machines and power engineering systems.

At the same time a lot of general questions of the theory of electromechanical systems have remained without attention. It is first of all the general problem of stability of equilibrium and motion of the electromechanical systems with linear and distributed currents, as well as the theory of periodic motions in electromechanical systems with a low electric dissipation. The priority belongs to K.Sh.Khodzhaev who is one of the authors of this book. The authors together with colleagues and post-graduate students have solved the important problems of simplification of the equations of synchronous electric machines by means of separating fast and slow electromechanical processes and some other problems of the qualitative analysis of their motions. K.Sh.Khodzhaev was the first who studied the oscillations of mechanical systems in the electromagnetic field which resulted in a series of engineering facilities.

The book consists of seven chapters. In the first chapter the basic features of the electromechanical systems are defined from positions of analytical mechanics. The theorems on motions in the electromechanical systems of various structures are formulated and proved.

In the second chapter the influence of a fast oscillating magnetic field on motion and equilibrium of the conducting rigid bodies is studied. Some particular problems presenting a certain interest for mechanical engineering are considered.

Some new simplified models for synchronous electric machines with various types of loadings are suggested in the third chapter. The qualitative dependences are studied and the general properties of motion of the electromechanical systems including the synchronous machines are determined.

The fourth chapter is concerned with formalization and further generalization of the problem of excitation of mechanical oscillations, in particular, in the electromechanical systems. Technical peculiarities of the electromechanical systems of vibration excitation allow one to formulate an integral criterion of stability of the periodic motions in these systems which differs from that in the theory of quasi-conservative synchronised systems [98].

In the fifth chapter the vibrations in mechanical systems caused by the electromagnetic vibration exciters of various types are studied in detail. The effects due to the interaction of processes in the exciter with the mechanical vibrations are discussed. The vibrations in the excitation systems with impacts and other types of mechanical nonlinearities are investigated. The influence of the magnetic nonlinearity on the vibration is determined.

The sixth chapter of the book is devoted to investigation of dynamics of relativistic and non-relativistic charged particle in fast oscillating and essentially inhomogeneous magnetic fields, this question being of a great importance for cosmophysics. Similar to the problems considered in the previous chapters this problem admits a correct and effective description and solution by means of the asymptotic methods of nonlinear mechanics. The main results are construction of higher approximations reduced to the equations of motion of a material point in the force field of a certain

structure or to the equations of motion in canonical form, the latter having less physical meaning however being more convenient for calculations and establishing the adiabatic invariants.

The seventh chapter deals with the statement and solving some “simplest” problems of the linear-elastic magnetoelasticity, i.e. the cases in which it is necessary to consider the dependence of the field and ponderomotive forces on the motion in spite of the fact that the latter can be rather small. The term “magnetoelasticity” has already been used in the works on elastic waves in conducting body and their interaction with the electromagnetic field. The described theory has a little in common with these problems since the present approach is concerned with the elastica, that is, the forms of elastic equilibrium of conducting or ferromagnetic rigid bodies in a constant field in which the waves are impossible. The obtained nonlinear boundary-value problems require some other methods of analysis.

Finally, new methods of solving the systems of differential equations with a small parameter contributing to development of the averaging method for some special cases are suggested and proved in the appendices.

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