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B. F. Shorr

The Wave Finite Element Method

With 164 Figures



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Preface

Computational mechanics, as a science employed for the numerical modeling of processes in nature and engineering, has over the last few decades developed two strands. The first concerns the putting of more and more powerful software packages into computational practice, using increasingly high-performance computers with increasingly large memory. The traditional finite element and finite difference approaches are still prevalent.

Over the years however, researchers have met with new problems; their solutions on the basis of traditional methods are at best difficult and at worst impossible to obtain. Such problems provided a powerful impetus in the development of the second strand, resulting in the development of essentially new approaches for numerical modeling, for example meshless methods, “molecular” dynamics, neuron networks. The current state of the art formed the basis of many papers presented at the Fifth World Congress on Computational Mechanics, Vienna 2002.

It is within the framework of the second strand that this book has been written.

The book concerns the problem of numerical finite element analysis of non-stationary dynamic processes in solids, using a novel ‘wave’ approach. The method is able to take into account strong discontinuities of stress, deformation, and velocity wave fronts, as well as finite wave propagation speeds. Such phenomena are particularly important within problems for which the time-scale is the same order as that of the time taken for a wave to propagate over the system; for instance, within explosions, shocks, seismic waves and structures with rapidly varying mass or stiffness.

The investigations were begun by the author in the mid 1970’s at the Central Institute of Aviation Engines (Moscow). The first results relating to 1-D problems were presented in a number of reports and journal papers. The methodology was applied to analysis of numerous engineering problems and generalized in a monograph by Shorr B.F and Mel’nikova G.V. “Calculation of Mechanical Systems Using the Method of Direct Mathematical Modelling”, 1988. However, all these publication, and other up until 1995, were in Russian and remain little known to western professionals.

In the last few years the ‘wave’ approach has been refined and extended to multi-dimensional processes, allowing a number of new solutions for unsteady problems in solids to be found. In this book the wave finite element method (WFEM) is systematically presented for the first time.

The book is designated for researchers, lecturers, post-graduates, and final year under-graduate students interested in problems of numerical modeling of non-stationary dynamic processes in deformable bodies and continua, as well as general problems of computational mechanics. Engineers and researchers, engaged in the design and analysis of machines and structures in which shock, vibro-impact and other unsteady dynamic processes play an important role may well find the book beneficial to their own research. The author believes that acquaintance with the method can be also of utility to specialists in design of electrical circuits and liquid currents in pipelines.

The author expresses deep gratitude to Prof. V.Babitsky, under whose initiative the English edition of the book was carried out and whose valuable councils the author repeatedly used, Prof. G.Rogerson, whose aid in ‘polishing’ the English of the book was very appreciated, and Dr. D.Merkle and the staff of Springer-Verlag for assistance in preparation and publication of the book. The author is also grateful to Prof. Ju.Kaplunov for useful discussion of a number of aspects of this work and to colleagues who have rendered help in realization of some numerical computations.

And finally, I am externally grateful to my wife and colleague Dr. G.Mel'nikova, without whose daily help and support this book would never have been ready for publication.

Boris Shorr, Moscow

Contents

Introduction	1
Theory	9
1 Foundation of the wave finite element method	11
1.1 Direct mathematical modeling of wave propagation in an elastic rod	11
1.1.1 Background equations	11
1.1.2 Numerical examples	20
1.2 Wave approach to finite element modeling	30
1.2.1 Background equations of the wave finite element method (WFEM).....	30
1.2.2 Numerical examples	38
2 Simulation of simple one-dimensional wave processes	43
2.1 Longitudinal waves in a rod	43
2.1.1 Collision of rods of different sizes and mechanical parameters	43
2.1.2 Sudden stopping of a rod of a variable cross section	48
2.1.3 Wave propagation in a rod with inner elastic-inertial links ...	50
2.2 Torsional waves in a rod	56
2.2.1 Sudden stopping of a rotating shaft	56
2.2.2 Setting a disk in motion by sudden connection with a rotating shaft	59
2.3 Transverse waves in strings and cables	61
2.3.1 Waves in a string stretched by a constant force	61
2.3.2 Waves in a cable stretched by its own weight	67
3 Wave propagation in an inelastic rod	73
3.1 Longitudinal waves propagation in an inelastic rod	73

3.1.1 Discrete-continual model of an inelastic rod	73
3.1.2 Governing equations	76
3.2 Waves in a viscoelastic rod.....	79
3.2.1 Background equations	79
3.2.2 Numerical examples	83
3.3 Waves in an elastic-viscoplastic rod	87
3.3.1 Elastic-plastic models	87
3.3.2 An elastic-viscoplastic model	94
4 Coupled longitudinal-torsional waves in a pre-twisted rod.....	97
4.1 Basic equations	97
4.1.1 Governing equations for a pre-twisted rod	97
4.1.2 Wave model of a pre-twisted rod	99
4.2 Wave propagation induced by a force and torque	103
4.2.1 Waves induced by a constant load	103
4.2.2 Impulse-induced waves	109
5 Bending waves in a beam.....	115
5.1 Basic equations	115
5.1.1 Wave model of the Timoshenko beam	115
5.1.2 Finite element simulation of bending waves	121
5.2 Direct mathematical modeling of bending waves propagation	126
5.2.1 Structural bending/shear model of a beam.....	126
5.2.2 Solution procedure	129
5.3 Numerical examples	131
5.3.1 A stepped force affecting a beam.....	131
5.3.2 A stepped moment affecting a beam.....	136
5.3.3 Comparison of the DMM and WFEM approaches for bending waves modeling	138
6 One-dimensional waves in elastic continua and structures.....	141
6.1 Plane waves	141
6.1.1 Longitudinal waves	141
6.1.2 Transverse and coupled waves	144
6.2 Spherical and cylindrical waves	147
6.2.1 Spherical waves	148
6.2.2 Explosion in a spherical cavity of an elastic medium	153

6.2.3 Cylindrical waves	155
7 Numerical simulation of multi-dimensional wave processes.....	159
7.1 Foundation of the general WFEM approach	159
7.1.1 Governing equations	159
7.1.2 Waves in a plane region. Code WPRD	166
7.2 Numerical examples	171
7.2.1 Sudden longitudinal loading of a one-side fixed plate	171
7.2.2 Sudden in-plane bending of a deep plate	175
7.2.3 A plate longitudinally impacted by a heavy body	177
7.2.4 A wide plate subjected to a bending moment	178
7.2.5 Additional remarks	181
Applications.....	187
8 Impact loading of a deformable body.....	189
8.1 Principle of floating boundary conditions (FBC)	189
8.1.1 Application of the FBC principle to WFEM	189
8.1.2 Special cases of body impact interaction	192
8.2 An elastic rod impacted by a rigid body	196
8.2.1 A rod of a constant cross section	196
8.2.2 The DMM accuracy in application to impact problems	200
8.2.3 A rod of variable cross section	204
8.3 An inelastic rod impacted by a rigid body	209
8.3.1 A rod of viscoelastic material	209
8.3.2 A rod of elastic-plastic material	210
8.4 Influence of contact deformation on impact response	213
8.4.1 Basic equations	213
8.4.2 Impact loading of a valve cylindrical spring	215
8.5 A pre-twisted rod impacted by a rigid body.....	217
8.5.1 Impact interaction of a rigid body with a pre-twisted rod ...	217
8.5.2 Lengthwise and turning impacts	220
9 Unsteady forced vibration of solids.....	225
9.1 Wave approach to study of forced vibration	225
9.1.1 Response of an elastic rod to harmonic excitation	225
9.1.2 Response of a rod of inelastic material.....	233
9.1.3 Transition through resonance domains under quasi-	

harmonic excitation	235
9.1.4 Response under fluctuating frequency and phase.....	238
9.2 Unsteady forced vibration of nonlinear systems.....	241
9.2.1 Torsional vibration of a shaft with a nonlinear clutch.....	241
9.2.2 Bending vibration of a turbine blade damped by a dry friction device	246
10 Unsteady vibro-impact loading.....	249
10.1 Multiple collisions at fixed points of a distributed system	249
10.1.1 Interaction of a rod with a viscoelastic foundation	249
10.1.2 Interaction of a rod with a hysteretic foundation	253
10.1.3 Switching on of a free-wheeling mechanism	256
10.2 Multiple collisions at varying points of a distributed system	260
10.2.1 Vibro-impact interaction of a string with limiters.....	260
10.2.2 A system with multiple inner gaps	262
11 Oscillations of a mechanical system affected by moving loads.....	265
11.1 General approach to simulation of moving loads	265
11.1.1 Equivalent node forces	265
11.1.2 Equivalent forces for different load/wave speeds ratio	268
11.2 Application of DMM to the study of 1-D waves induced by moving loads.....	271
11.2.1 A strip on a viscoelastic foundation	271
11.2.2 A beam on a viscoelastic foundation.....	274
11.3 Application of WFEM to the study of 2-D waves induced by moving loads.....	278
11.3.1 A long plate loaded by a transverse moving force	278
11.3.2 A long plate loaded by a longitudinal moving force	281
12 Dynamic loading of a free edge of a solid.....	285
12.1 Constant loads suddenly affecting a thin plate	285
12.1.1 A point force	285
12.1.2 A distributed load	289
12.2 Varying loads affecting a half-space	290
12.2.1 A point impulsive force	290
12.2.2 A distributed impulsive load applied to a limited domain	296

13 Some special problems of solid mechanics.....299

 13.1 Deformation of a chain of a varying length.....299

 13.1.1 Sliding down of an elastic chain under own weight299

 13.1.2 Numerical example302

 13.2 Waves in structures interacting with ‘active’ media304

 13.2.1 Strings on an ‘anti-elastic’ or ‘anti-viscous’ foundation ..305

 13.2.2 Auto-oscillation of a string in nonlinear viscous
 medium.....308

 13.2.3 Auto-oscillation in a system with intermittent contacts...311

14 Some special unsteady problems in engineering.....315

 14.1 Longitudinal dynamics of a train315

 14.1.1 Setting of a problem315

 14.1.2 Transient regimes of a train motion317

 14.2 Wave problems in adjacent areas of engineering319

 14.2.1 A transient process in an electrical circuit320

 14.2.2 Unsteady hydraulics problems324

Conclusion.....329

References.....331

Appendix.....337