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L.I. Manevitch · I.V. Andrianov · V.G. Oshmyan, Mechanics of Periodically Heterogeneous Structures

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Mechanics of Periodically Heterogeneous Structures

With 76 Figures

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Preface

Relieve the problem of inessential detail
and reduce it to the simplest elements.
Rene Descartes

Heterogeneity of a material or construction can be caused by two main reasons. Non-uniformity of certain physical characteristics (density, elastic modulus, conductivity, etc.) is the first. Two- or multi-phase composites are typical examples of this type of material. The second origin of heterogeneity is a geometrical one. Reinforcement of the shells and plates by stringers, discrete supports and other constructive elements is widely used in numerous applications. Both reasons cause heterogeneity of the stress–strain state and the descriptions of the mechanical responses meet very similar mathematical difficulties. Therefore, it is natural to analyze and solve the corresponding boundary-value problems applying a similar and in some cases identical technique.

Many problems in modern composites, heterogeneous plates and shells theory are governed by partial differential equations with rapidly changing and mostly discontinuous coefficients. Obviously, there are two opposite limits in which the direct application of conventional technique is efficient.

The first limit is a small number of heterogeneities. It means that the scale, l, of inhomogeneity (inclusion diameter, distance between stringers, etc.) is of the same order as the typical outer size, L, of the structure, $L \propto l$. Direct numerical methods (finite elements, finite differences, etc.) should be applied in this case. The high level of modern digital computing power provides precise results in numerous complicated problems of composites, plates and shell mechanics.

However, even modern computers [65,71,158] cannot efficiently assist in solving the problems corresponding to the mechanics of heterogeneous media in the opposite limit $L >> l$. This is a reason for an application of certain kind of homogenization technique in this limit: effective media theory (EMT, the term used in composite mechanics) or structurally orthotropic theory (SOT, the term used in plate and shell mechanics) in particular. The replacement of heterogeneous media by the homogeneous continuum, which is characterized by certain effective constitutive equations, is the basic instrument for both EMT and SOT. Four important questions should be resolved in the context of homogenization of the media.

(1) Clearly, the limit $L/l \to \infty$ is a necessary condition for the possibility of correct approximation of heterogeneous media by the homogeneous one. However, it is not clear at all whether this condition is sufficient. Numerous mathematical studies (mathematical homogenization theory, MHT) are devoted to this subject. The

rigorous proof of the existence of the homogenization limit is the main aim of corresponding publications and very important results have been obtained in this field.

(2) Let us suppose the possibility of the homogenization. The next problem then arises: how to determine the effective constitutive equations (composite elastic moduli, particularly). Direct averaging is obviously wrong, but is often applied for homogenization. Effective values of constitutive constants can be bounded with the help of various variational theorems. Unfortunately, the bounds obtained are very wide and cannot be improved in the practically important case of a sharp difference in component properties. Self-consistent approaches are popular and efficient techniques for the estimation of effective elastic constants. The rigorous MHT reduces the calculation of the effective properties to the solution of boundary-value problems for the periodicity cell in the case of a periodic structure, which allows application of conventional numerical techniques.

(3) Effective medium theories are able to describe the main terms of displacement fields only, but not the local stress–strain state. Corresponding information has to be determined if one is going to estimate fracture parameters.

(4) Numerous important applications (especially of plate and shell theory) deal with the case of a finite *L/l* ratio. In this intermediate region, the effective uniform description is obviously insufficient and high-order asymptotic expansions should be determined.

Rigorous presentation of MHT is the subject of numerous papers, reviews and manuscripts, and we do not pursue the goal of extending and improving this account. However, we see a gap in the analytical and/or numerical performance of the corresponding asymptotic analysis of heterogeneous system static and dynamic behaviors. The main purpose of this book is to fill this gap. Numerous applications to composite media, heterogeneous plates and shells are considered. We include a lot of details, numerical results for cell problem solutions, calculations of high-order terms of asymptotic expansions, boundary layer analysis, etc. The representation is mainly based on the original results of the authors in collaboration with A. Diskovsky, A. Givental, L. Givental, M. Guy, E. Kholod, N. Knunyantz, S. Koblik, N. Kozhina, G. Krizhevsky, V. Lesnichaya, V. Loboda, A. Pavlenko, A. Pisanko, A. Shamrovskii, V. Shevchenko, G. Starushenko and S. Timan.

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