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Geometric Aspects of Convex Sets with the Radon-Nikodým Property

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To Leighton

PREFACE

Johann Radon [1913] established the theorem which bears his and Nikodým's name for measures of finite total variation on the Borel σ-field of Euclidean n-space; Otton Nikodým [1930] removed what are now known to be the unnecessarily restrictive assumptions Radon had imposed. Only three years elapsed from Nikodým's general theorem to the first published accounts dealing with the problem of differentiating Banach-valued absolutely continuous functions on [0,1]. There followed a spate of geometrical Banach space developments in the middle 1930s motivated by the search for conditions guaranteeing differentiability, the best known being Clarkson's introduction of uniformly convex spaces in 1936. After the results of Dunford, Pettis and Phillips in 1940 on the representability of linear operators on $L^1(\mu)$ as Bochner integrals, however, few direct substantive advances on Radon-Nikodým issues in Banach spaces were noted until the late 1960s. At that time M.A. Rieffel tied the Radon-Nikodým theorem in Banach spaces to the geometry via the notion of dentability. Also, S.D. Chatterji spelled out the intimate relationship between convergence of L^1-bounded Banach-valued martingales and the existence of Bochner integrable Radon-Nikodým derivatives. These results, in tandem with the concurrent geometrical advances of Lindenstrauss and others on the existence of extreme points for not necessarily compact closed bounded convex subsets of certain Banach spaces, spawned considerable research activity. A coherent picture of Banach spaces having the "Radon-Nikodým Property" developed from operator, martingale and geometric perspectives.

By the mid 1970s the broad strokes were in place and a period of reevaluation and refinement began. One of the upshots was the localization of many of the early theorems on Banach spaces to the corresponding ones for closed bounded convex sets. A second, expected improvement, was the tightening and reworking of several of the original arguments. What emerged was an elegant and comprehensive theory of sets with the Radon-Nikodým Property. In Chapters 2 through 6 of these notes this theory is described in detail, special attention being given to the geometrical aspects. By the late 1970s the Radon-Nikodým novelty had died down as the theory of such sets was by then fairly well understood. Emphasis shifted from the study of sets with the "RNP", which until then had flourished, to an analysis of their place in the larger functional analytic picture, especially their role in the structure theory of Banach spaces. The second half of these notes, Chapter 7, is a report on some aspects of this very active phase of the continuing Radon-Nikodým saga. Most of the topics discussed in Chapter 7, as listed in the Table of Contents, may be read independently of one another. (The parenthetical comments in the Table of Contents most often indicate subjects, in addition to those mentioned in section titles, which have been broached in the text. They are not to be construed as a full listing of the contents.)

I wish to thank each of the many people who has contributed in one way or another to this work. The notes would not have been completed without the timely receipt of manuscripts just written, the long discussions and the correspondence I have had on its contents. Several people provided detailed suggestions about portions of the first six chapters, many of which have been incorporated in the final version. They include G.A. Edgar, B. Epstein, S. Fitzpatrick and R.R. Phelps, to each of whom I extend my appreciation. Throughout this project I have had the benefit of the wisdom, advice, thoughtful comments and careful and elegant mathematical arguments of Isaac Namioka. His support has contributed significantly to making the work on these notes enjoyable. I am in his debt.

It is intended that anyone with a standard background in measure theory and basic functional analysis could, with a minimum of additional background as briefly outlined in the first part of Chapter 1, read large portions of these notes without undue difficulty. Arguments are presented in detail and, especially in the early chapters, some attempt has been made to predict later results on the basis of examples and preliminary theorems. A good example is afforded by Lévy's theorem in §1.3 which marks the dividing line between general martingale arguments substantively independent of the range space and those martingale arguments in which a knowledge of the range space is essential. The study of sets with the Radon-Nikodým Property begins in earnest in Chapter 2; the basic connections between martingale, measure theoretic and geometric notions is established by the end of §2.2. The sharpest geometric conclusions in these notes may be found in §§3.5, 3.6 and 3.7. Each of the next three chapters deals with a subtopic worthy of extended discussion. Although the Radon-Nikodým Property is central to each of the sections of Chapter 7, the emphasis is often different than in the earlier chapters. In many of the sections, rather than discovering more about the structure of such sets, the RNP is used as a tool to study other objects. By virtue of this distinction, the range of background required for some of the sections of the last chapter is greater than that for the first six chapters, and in such cases I have provided references for additional background material. I have drawn on numerous sources, most of which have been credited in the text. The 1976-77 Rainwater seminar notes from the University of Washington were inadvertantly not credited in the text. They were used extensively while writing portions of Chapters 3 and 4.

Most notation is standard. However, it is worthwhile to mention here the use of "$\equiv$" in the text to denote equality in which one of the terms is defined by the equality. Thus, for example, $\mathbb{N}^+ \equiv \{1,2,3...\}$. With very few exceptions, discussions are limited to real Banach spaces and their subsets. Unless it is explicitly mentioned to the contrary, the word "subspace" means "closed linear subspace" in these notes.

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