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Edited by A.V. Balakrishnan and M. Thoma

40

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Multivariable Feedback:
A Quasi-Classical Approach



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CONTENTS

NOTATION		iii
CHAPTER 0	INTRODUCTION	1
CHAPTER 1	SINGULAR-VALUE, CHARACTERISTIC-VALUE AND POLAR DECOMPOSITIONS	5
1.1	System Description and Feedback Configurations	5
1.2	Characteristic Gain Loci and the Generalized Nyquist Stability Criterion	7
1.3	The Singular Value Decomposition (SVD)	10
1.4	SVD of a Continuous Matrix-Valued Function	13
1.5	Polar Decomposition (PD)	15
1.6	Normality and Spectral Sensitivity	17
CHAPTER 2	USE OF PARAMETER GROUP DECOMPOSITION TO GENERATE NYQUIST-TYPE LOCI	21
2.1	Some Matrix Groups and their Parametrizations	22
2.2	Dimension of Matrix Groups	26
2.3	Nyquist-Type Loci — the PG Loci	27
2.4	Relationship between the Parameter Group Decomposition and Normality	32
2.5	Parametrization of Higher Order Matrix Groups	34
2.6	A Drawback of the Parameter Group Decomposition	35
CHAPTER 3	ALIGNMENT, NORMALITY AND QUASI-NYQUIST LOCI	37
3.1	Frame Alignment and Normality	37
3.2	Relationship between Skewness and Misalignment	42
3.3	The Quasi-Nyquist Decomposition (QND)	42
3.4	Eigenvalue Bounds and the QND	44
3.5	Quasi-Nyquist Loci (QNL)	49
3.6	Standardization at $s=0$ or ∞	53
	3.6.1 Standardization at $s=0$	53
	3.6.2 Standardization at $s=\infty$	54
3.7	Diagonalizing at a Critical Frequency	57
CHAPTER 4	A QUASI-CLASSICAL DESIGN TECHNIQUE	66
4.1	Computer-Aided Control System Design	66
4.2	Stability	68

4.3	Performance	69
4.3.1	Reversed-Frame-Normalizing (RFN) Controller	70
4.3.2	Interaction	72
4.3.3	Tracking Accuracy and Disturbance Rejection	74
4.4	Robustness	75
4.5	Robustness and Reversed-Frame-Normalization (RFN)	80
4.6	Compatibility Conditions	82
4.7	Specifying a Desired Compensated System	85
CHAPTER 5	CALCULATING A COMPENSATOR NUMERATOR MATRIX BY LINEAR LEAST-SQUARES FITTING	90
5.1	Reversed-Frame-Normalizing Design Procedure (RFNDP)	90
5.2	Some Results for the Linear Least-Squares Problem	93
5.3	Calculation of the Precompensator Numerator Matrix	96
5.4	Example	98
CHAPTER 6	CALCULATING A COMPENSATOR BY NONLINEAR LEAST-SQUARES FITTING	105
6.1	Problem Formulation	106
6.2	A Least-Squares Problem whose Variables Separate	109
6.3	Example	112
CHAPTER 7	EXAMPLES OF THE DESIGN TECHNIQUES	117
7.1	A Design Example for a Turbo-Generator	117
7.2	Non-Square Systems	124
7.2.1	Systems with More Inputs than Outputs	124
7.2.2	Systems with More Outputs than Inputs	126
7.3	Design Examples for Systems with More Outputs than Inputs	131
7.4	General Conclusion	148
APPENDIX A	Analytic Properties of the Singular Values of a Rational Matrix	150
APPENDIX B	Proofs of Prop 3.2.1, Prop 3.3.1, Prop 4.5.1 and Theorem 4.6.2	156
APPENDIX C	The System AUTM	163
APPENDIX D	The Systems NSRE and REAC	165
APPENDIX E	The System TGEN	167
APPENDIX F	The System AIRC	169
REFERENCES		171
BIBLIOGRAPHY		176
INDEX		180

NOTATION

A list of recurrent symbols is given below.

$a := b$ means a is defined to be b or a denotes b

\mathbb{R}, \mathbb{C} := field of real and complex numbers, respectively

\mathbb{C}_+ := $\{z \in \mathbb{C} \mid \operatorname{Re} z \geq 0\}$, the closed right half plane (closed RHP)

\mathbb{C}_- := $\{z \in \mathbb{C} \mid \operatorname{Re} z \leq 0\}$, the closed left half plane (closed LHP)

\mathbb{C}_+^* := $\mathbb{C}_+ - \{0\}$

$D(c; r)$:= $\{z \in \mathbb{C} \mid |z - c| \leq r\}$, the closed disc centre c , radius r

For any $\Omega \subset \mathbb{C}$,

Ω° := interior of Ω , e.g. \mathbb{C}_+° denotes the open RHP

For $z \in \mathbb{C}$

$|z|$:= modulus (or magnitude) of z

$\angle z, \arg z$:= argument of z

$\operatorname{Re} z, \operatorname{Im} z$:= real, imaginary part of z , respectively

\bar{z} := complex conjugate of z

$\mathbb{R}[s]$:= ring of polynomials in s with coefficients in \mathbb{R}

$\mathbb{R}(s), \mathbb{C}(s)$:= field of rational functions in s with coefficients in \mathbb{R}, \mathbb{C}

$\mathbb{R}_p(s)$:= $\{g(s) \in \mathbb{R}(s) \mid \lim_{s \rightarrow \infty} |g(s)| < \infty\}$, set of proper rational functions

$\mathbb{R}_{sp}(s)$:= $\{g(s) \in \mathbb{R}(s) \mid \lim_{s \rightarrow \infty} |g(s)| = 0\}$, set of strictly proper rational functions

Let \mathbb{F} be any one of $\mathbb{R}, \mathbb{C}, \mathbb{R}[s], \mathbb{R}(s), \mathbb{C}(s), \mathbb{R}_p(s)$ or $\mathbb{R}_{sp}(s)$, then:

$\mathbb{F}^{m \times \ell}$:= set of $m \times \ell$ matrices with elements in \mathbb{F}

\mathbb{F}^n := vector space of $n \times 1$ column vectors with elements in \mathbb{F} , over an appropriate field

Let $M \in \mathbb{F}^{m \times \ell}$ where $\mathbb{F} = \mathbb{R}$ or \mathbb{C} , then:

m_{ij} := (i, j) th entry of M ; we also write $M = (m_{ij})$

$\lambda(M)$:= spectrum (set of eigenvalues) of M

$\sigma(M)$:= set of singular values of M

$\sigma_{\max}(M)$:= $\max \sigma(M)$, maximum singular value of M

$\sigma_{\min}(M)$:= $\min \sigma(M)$, minimum singular value of M

M^T := transpose of M

M^* := conjugate transpose of M

M^\dagger := Moore-Penrose inverse of M

$|M|$:= (x_{ij}) where $x_{ij} = |m_{ij}|$

$\arg M$:= (x_{ij}) where $x_{ij} = \arg m_{ij}$

$\mathcal{R}(M)$:= $\{Mx \mid x \in \mathbb{F}^\ell\}$, range space of columns of M

$\mathcal{N}(M)$:= $\{x \in \mathbb{F}^\ell \mid Mx = 0\}$, right null space of M

$\text{Tr}(M)$:= $\sum_{i=1}^m m_{ii}$, trace of M , if M is square

$\|M\|$:= $[\text{Tr}(M^*M)]^{1/2} = \left(\sum_{j=1}^{\ell} \sum_{i=1}^m |m_{ij}|^2\right)^{1/2}$, Frobenius norm of M

$\|M\|_2$:= $\sigma_{\max}(M)$, spectral norm of M

For any $W \in \mathbb{F}^{m \times \ell}$, we define the weighted Frobenius norm by weighting elementwise:

$\|M\|_W$:= $\left(\sum_{j=1}^{\ell} \sum_{i=1}^m |w_{ij}| |m_{ij}|^2\right)^{1/2}$

For any other matrix $N \in \mathbb{F}^{r \times s}$

$M \otimes N$:= $\begin{bmatrix} m_{11}N & \dots & m_{1m}N \\ \dots & \dots & \dots \\ m_{\ell 1}N & \dots & m_{\ell m}N \end{bmatrix}$, the Kronecker product of M and N

I_m := $m \times m$ unit matrix

$\mathbb{1}_m$:= $m \times m$ matrix filled with 1's

Let $u \in \mathbb{F}^\ell$ where $\mathbb{F} = \mathbb{R}$ or \mathbb{C} , then:

$\|u\|$:= $(u^*u)^{1/2} = \left(\sum_{i=1}^{\ell} |u_{ij}|^2\right)^{1/2}$, the Euclidean vector norm

Let $W = P^*P \in \mathbb{C}^{\ell \times \ell}$ be a hermitian, positive definite (weighting) matrix,

$\|u\|_W$:= $\|Pu\| = (u^*Wu)^{1/2}$, the weighted Euclidean vector norm

Let columns of $V \in \mathbb{F}^{t \times \ell}$ ($t < \ell$) be a basis of a subspace of \mathbb{F}^ℓ , then:

V^\perp $\in \mathbb{F}^{(\ell-t) \times \ell}$ and its columns form a basis for the orthogonal complement of $\mathcal{R}(V)$

P_V := VV^\dagger , the orthogonal projector onto $\mathcal{R}(V)$

P_V^\perp := $I - P_V = V^\perp V^{\perp \dagger}$, the orthogonal projector onto $\mathcal{R}(V^\perp)$

For $p(s) \in \mathbb{R}[s]$, $P(s) \in \mathbb{R}[s]^{m \times l}$

$\deg p(s) \quad :=$ degree of the polynomial $p(s)$

$\deg[\text{row}_i(P(s))] \quad :=$ max degree of the polynomials in the i th row of $P(s)$

$\text{diag}(d_i)_{i=1}^n \quad :=$ $n \times n$ diagonal matrix with d_1, \dots, d_n along the diagonal; also written as $\text{diag}(d_1, \dots, d_n)$ or $\text{diag}(d_i)$

$p\text{-diag}(d_i)_{i=1}^n \quad :=$ pseudo-diagonal matrix with d_1, \dots, d_n along its principal diagonal

Let $\Omega \subset \mathbb{C}$, $f(s) \in \mathbb{R}(s)$ and $G(s) \in \mathbb{R}(s)^{m \times l}$, then:

$\#Z(f(s), \Omega) \quad :=$ number of zeros (multiplicities counted) of $f(s)$ in Ω

$\#P(f(s), \Omega) \quad :=$ number of poles (multiplicities counted) of $f(s)$ in Ω

$\#\text{SMZ}(G(s), \Omega) \quad :=$ number of Smith-McMillan zeros of $G(s)$ in Ω

$\#\text{SMP}(G(s), \Omega) \quad :=$ number of Smith-McMillan poles of $G(s)$ in Ω

$\#\text{IZ}(G(s)) \quad :=$ number of ∞ zeros (multiplicities counted) of $G(s)$

Let γ be a (finite number of) closed curve(s) in \mathbb{C} , then:

$\#E(\gamma, a) \quad :=$ number of encirclements of γ around the point a
(our convention is positive for anticlockwise)

D_{NYQ} Nyquist D-contour, see §1.2

$\text{MS}(\cdot)$ measure of skewness, see §1.6

$\text{GL}(n, \mathbb{C})$ general linear group, see §2.1

$U(n)$ unitary group, see §2.1

$SU(n)$ special unitary group, see §2.1

$m(G)$ frame misalignment of G , see §3.1

$\text{TPC}(G(s))$ total phase change of the characteristic gain loci of $G(s)$, see §4.6

fog denotes the composition of two functions, f after g

\forall, \exists denotes for all, there exist(s)

\square marks the end or the absence of a proof

List of Abbreviations:

AIRC	Aircraft Dynamics Model, Appendix F
AUTM	Automobile Gas Turbine Model, Appendix C
CAD	Computer-Aided-Design, §4.1
CGL, CGLi	Characteristic Gain Loci, ith branch of, §1.2
CVD	Characteristic Value Decomposition
CLTM	Closed-Loop Transfer Matrix, §4.3.2
GMI	Gain Margin Interval, §4.4
LHP	Left Half Plane
LQR	Linear Quadratic Regulator
MFD	Matrix Fraction Description
NSRE	Non-Square Chemical Reactor Model, Appendix D
PD	Polar Decomposition, §1.5
PGD	Parameter Group Decomposition, §2.1
PGL, PGLi	Parameter Group Loci, ith branch of, §2.3
PI	Proportional plus Integral
PMI	Phase Margin Interval, §4.4
QND	Quasi-Nyquist Decomposition, §3.2
QNL, QNLi	Quasi-Nyquist Loci, ith branch of, §3.5
REAC	Chemical Reactor Model, Appendix D
RFN	Reversed-Frame Normalizing/Normalization
RFNDP	Reversed-Frame Normalizing Design Procedure, §5.1
RHP	Right Half Plane
SVD	Singular Value Decomposition, §1.3
STD	Schur Triangular Decomposition, §1.6
s.t.	such that
TGEN	Turbo-Generator Model, Appendix E
w.r.t.	with respect to