# Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

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Y.S. Hung
A.G.J. MacFarlane


# Multivariable Feedback: A Quasi-Classical Approach


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**Authors**
Y. S. Hung · A. G. J. MacFarlane
Engineering Department
University of Cambridge
Mill Lane
Cambridge CB2 1RX
England

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Y.S.H and A.G.J.M

# CONTENTS

## NOTATION

A list of recurrent symbols is given below.

$a := b$ means a is defined to be b or a denotes b

$\mathbb{R}$, $\mathbb{C}$ := field of real and complex numbers, respectively

$\mathbb{C}_+$ := $\{z \in \mathbb{C} \mid \operatorname{Re} z \geq 0\}$, the closed right half plane (closed RHP)

$\mathbb{C}_-$ := $\{z \in \mathbb{C} \mid \operatorname{Re} z \leq 0\}$, the closed left half plane (closed LHP)

$\mathbb{C}_+^*$ := $\mathbb{C}_+ - \{0\}$

$D(c;r)$ := $\{z \in \mathbb{C} \mid |z-c| \leq r\}$, the closed disc centre c, radius r

For any $\Omega \subset \mathbb{C}$,

$\Omega^\circ$ := interior of $\Omega$, e.g. $\mathbb{C}_+^\circ$ denotes the open RHP

For $z \in \mathbb{C}$

$|z|$ := modulus (or magnitude) of z

$\angle z$, $\arg z$ := argument of z

$\operatorname{Re} z$, $\operatorname{Im} z$ := real, imaginary part of z, respectively

$\bar{z}$ := complex conjugate of z

$\mathbb{R}[s]$ := ring of polynomials in s with coefficients in $\mathbb{R}$

$\mathbb{R}(s)$, $\mathbb{C}(s)$ := field of rational functions in s with coefficients in $\mathbb{R}$, $\mathbb{C}$

$\mathbb{R}_p(s)$ := $\{g(s) \in \mathbb{R}(s) \mid \lim_{s\to\infty} |g(s)| < \infty\}$, set of proper rational functions

$\mathbb{R}_{sp}(s)$ := $\{g(s) \in \mathbb{R}(s) \mid \lim_{s\to\infty} |g(s)| = 0\}$, set of strictly proper rational functions

Let $\mathbb{F}$ be any one of $\mathbb{R}$, $\mathbb{C}$, $\mathbb{R}[s]$, $\mathbb{R}(s)$, $\mathbb{C}(s)$, $\mathbb{R}_p(s)$ or $\mathbb{R}_{sp}(s)$, then:

$\mathbb{F}^{m\times\ell}$ := set of $m\times\ell$ matrices with elements in $\mathbb{F}$

$\mathbb{F}^n$ := vector space of $n\times 1$ column vectors with elements in $\mathbb{F}$, over an appropriate field

Let $M \in \mathbb{F}^{m\times\ell}$ where $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$, then:

$m_{ij}$ := (i,j)th entry of M; we also write $M = (m_{ij})$

$\lambda(M)$ := spectrum (set of eigenvalues) of M

$\sigma(M)$ := set of singular values of M

$\sigma_{max}(M)$ := $\max \sigma(M)$, maximum singular value of M

$\sigma_{min}(M)$ := $\min \sigma(M)$, minimum singular value of M

$M^T$ := transpose of M

$M^*$ := conjugate transpose of M

$M^\dagger$ := Moore-Penrose inverse of M

$|M|$ := $(x_{ij})$ where $x_{ij} = |m_{ij}|$

arg M := $(x_{ij})$ where $x_{ij} = \arg m_{ij}$

$\mathscr{R}(M)$ := $\{Mx \mid x \in \mathbb{F}^\ell\}$, range space of columns of M

$\mathscr{N}(M)$ := $\{x \in \mathbb{F}^\ell \mid Mx = 0\}$, right null space of M

Tr(M) := $\sum_{i=1}^{m} m_{ii}$ , trace of M, if M is square

$\|M\|$ := $[\mathrm{Tr}(M^*M)]^{1/2} = (\sum_{j=1}^{\ell} \sum_{i=1}^{m} |m_{ij}|^2)^{1/2}$, Frobenius norm of M

$\|M\|_2$ := $\sigma_{max}(M)$, spectral norm of M

For any $W \in \mathbb{F}^{m\times\ell}$, we define the weighted Frobenius norm by weighting elementwise:

$\|M\|_W$ := $(\sum_{j=1}^{\ell} \sum_{i=1}^{m} |w_{ij}|\,|m_{ij}|^2)^{1/2}$

For any other matrix $N \in \mathbb{F}^{r\times s}$

$$M \otimes N := \begin{bmatrix} m_{11}N & \cdots & m_{1m}N \\ \cdots & \cdots & \cdots \\ m_{\ell 1}N & \cdots & m_{\ell m}N \end{bmatrix},$$ the Kronecker product of M and N

$I_m$ := m×m unit matrix

$\mathbb{1}_m$ := m×m matrix filled with 1's

Let $u \in \mathbb{F}^\ell$ where $\mathbb{F} = \mathbb{R}$ or $\mathbb{C}$, then:

$\|u\|$ := $(u^*u)^{1/2} = (\sum_{i=1}^{\ell} |u_{ij}|^2)^{1/2}$, the Euclidean vector norm

Let $W = P^*P \in \mathbb{C}^{\ell\times\ell}$ be a hermitian, positive definite (weighting) matrix,

$\|u\|_W$ := $\|Pu\| = (u^*Wu)^{1/2}$, the weighted Euclidean vector norm

Let columns of $V \in \mathbb{F}^{t\times\ell}$ $(t < \ell)$ be a basis of a subspace of $\mathbb{F}^\ell$, then:

$V^\perp$ $\in \mathbb{F}^{(\ell-t)\times\ell}$ and its columns form a basis for the orthogonal complement of $\mathscr{R}(V)$

$P_V$ := $VV^\dagger$, the orthogonal projector onto $\mathscr{R}(V)$

$P_V^\perp$ := $I - P_V = V^\perp V^{\perp\dagger}$, the orthogonal projector onto $\mathscr{R}(V^\perp)$

For $p(s) \in \mathbb{R}[s]$, $P(s) \in \mathbb{R}[s]^{m\times \ell}$

$\deg p(s)$ := degree of the polynomial $p(s)$

$\deg[\mathrm{row}_i(P(s))]$ := max degree of the polynomials in the $i$th row of $P(s)$

$\mathrm{diag}(d_i)_{i=1}^n$ := $n\times n$ diagonal matrix with $d_1,\ldots,d_n$ along the diagonal; also written as $\mathrm{diag}(d_1,\ldots,d_n)$ or $\mathrm{diag}(d_i)$

$\text{p-diag}(d_i)_{i=1}^n$ := pseudo-diagonal matrix with $d_1,\ldots,d_n$ along its principal diagonal

Let $\Omega \subset \mathbb{C}$, $f(s) \in \mathbb{R}(s)$ and $G(s) \in \mathbb{R}(s)^{m\times \ell}$, then:

$\#Z(f(s),\Omega)$ := number of zeros (multiplicities counted) of $f(s)$ in $\Omega$

$\#P(f(s),\Omega)$ := number of poles (multiplicities counted) of $f(s)$ in $\Omega$

$\#SMZ(G(s),\Omega)$ := number of Smith-McMillan zeros of $G(s)$ in $\Omega$

$\#SMP(G(s),\Omega)$ := number of Smith-McMillan poles of $G(s)$ in $\Omega$

$\#IZ(G(s))$ := number of $\infty$ zeros (multiplicities counted) of $G(s)$

Let $\gamma$ be a (finite number of) closed curve(s) in $\mathbb{C}$, then:

$\#E(\gamma,a)$ := number of encirclements of $\gamma$ around the point $a$ (our convention is positive for anticlockwise)

| | |
|---|---|
| $D_{NYQ}$ | Nyquist D-contour, see §1.2 |
| $MS(\cdot)$ | measure of skewness, see §1.6 |
| $GL(n,\mathbb{C})$ | general linear group, see §2.1 |
| $U(n)$ | unitary group, see §2.1 |
| $SU(n)$ | special unitary group, see §2.1 |
| $m(G)$ | frame misalignment of G, see §3.1 |
| $TPC(G(s))$ | total phase change of the characteristic gain loci of G(s), see §4.6 |

| | |
|---|---|
| $f \circ g$ | denotes the composition of two functions, f after g |
| $\forall$ , $\exists$ | denotes for all, there exist(s) |
| $\square$ | marks the end or the absence of a proof |

List of Abbreviations:

| | |
|---|---|
| AIRC | Aircraft Dynamics Model, Appendix F |
| AUTM | Automobile Gas Turbine Model, Appendix C |
| CAD | Computer-Aided-Design, §4.1 |
| CGL, CGLi | Characteristic Gain Loci, ith branch of, §1.2 |
| CVD | Characteristic Value Decomposition |
| CLTM | Closed-Loop Transfer Matrix, §4.3.2 |
| GMI | Gain Margin Interval, §4.4 |
| LHP | Left Half Plane |
| LQR | Linear Quadratic Regulator |
| MFD | Matrix Fraction Description |
| NSRE | Non-Square Chemical Reactor Model, Appendix D |
| PD | Polar Decomposition, §1.5 |
| PGD | Parameter Group Decomposition, §2.1 |
| PGL, PGLi | Parameter Group Loci, ith branch of, §2.3 |
| PI | Proportional plus Integral |
| PMI | Phase Margin Interval, §4.4 |
| QND | Quasi-Nyquist Decomposition, §3.2 |
| QNL, QNLi | Quasi-Nyquist Loci, ith branch of, §3.5 |
| REAC | Chemical Reactor Model, Appendix D |
| RFN | Reversed-Frame Normalizing/Normalization |
| RFNDP | Reversed-Frame Normalizing Design Procedure, §5.1 |
| RHP | Right Half Plane |
| SVD | Singular Value Decomposition, §1.3 |
| STD | Schur Triangular Decomposition, §1.6 |
| s.t. | such that |
| TGEN | Turbo-Generator Model, Appendix E |
| w.r.t. | with respect to |