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## **Editors**

Jaurés P. Cecconi

Tullio Zolezzi

Istituto per la Matematica Applicata CNR

Presso Istituto Matematico, Università di Genova

Via L.B. Alberti 4, Genova, Italia

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Most of the papers were presented at this conference and a few others were contributed by mathematicians unable to attend.

The conference aimed at up-to-date contributions and surveys in mathematical optimizations. The research papers of these proceedings cover many subjects, from optimal control to calculus of variations, from operations research to non smooth optimization and Gamma Convergence.

The meeting brought together many mathematicians from many countries who represented a wide range of interests in the subject, and was attended by approximately eighty mathematicians.

We use this opportunity to thank the members of the Istituto per la Matematica del C.N.R. di Genova for their support in the organization. We wish to express our thanks to the institutions whose financial support made the conference possible. These are Consiglio Nazionale delle Ricerche, Istituto di Matematica della Università di Genova and Regione Liguria.

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J. P. Cecconi

T. Zolezzi

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