

Lecture Notes in Control and Information Sciences

Edited by A.V. Balakrishnan and M. Thoma

65

Yaakov Yavin

Numerical Studies
in Nonlinear Filtering



Springer-Verlag
Berlin Heidelberg New York Tokyo

Series Editors

A. V. Balakrishnan · M. Thoma

Advisory Board

L. D. Davissou · A. G. J. MacFarlane · H. Kwakernaak

J. L. Massey · Ya Z. Tsypkin · A. J. Viterbi

Author

Yaakov Yavin

c/o NRIMS

CSIR

P.O. Box 395

Pretoria 0001 – South Africa

ISBN 3-540-13958-3 Springer-Verlag Berlin Heidelberg New York Tokyo

ISBN 0-387-13958-3 Springer-Verlag New York Heidelberg Berlin Tokyo

Library of Congress Cataloging in Publication Data

Yavin, Yaakov

Numerical studies in nonlinear filtering.

(Lecture notes in control and information sciences; 65)

Includes bibliographies.

1. System analysis.

2. Filters (Mathematics).

3. Estimation theory.

I. Title.

II. Series.

QA402.Y3788 1985 003 84-23567

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

© Springer-Verlag Berlin, Heidelberg 1985

Printed in Germany

Offsetprinting: Color-Druck, G. Baucke, Berlin

Binding: Lüderitz und Bauer, Berlin

2061/3020-543210

PREFACE

State estimation techniques were developed for situations in engineering in which, based on nonlinear and noise-corrupted measurements of a process, and on a good model of the process, the process is estimated either on-line or off-line using the available measurements.

These techniques became known in the early sixties under the celebrated name of Kalman filtering, and were applied mainly to linear problems. Later developments by Kushner, Wonham and others led to solutions to nonlinear state estimation problems, in which, in general, infinite-dimensional filters are required. Practical algorithms, such as the linearized and extended Kalman filters, which involve only finite-dimensional filters, have been most frequently used as approximate solutions to these nonlinear state estimation problems.

The present work offers some new approaches to the construction of finite-dimensional filters as approximate solutions to nonlinear state estimation problems. Numerical procedures for the implementation of these filters are given, and the efficiency and applicability of these procedures is demonstrated by means of numerical experimentation.

It is my pleasant duty to record here my sincere thanks to the National Research Institute for Mathematical Sciences of the CSIR for encouraging this research. I gratefully acknowledge the contribution made by Mrs. H C Marais, Mrs. R de Villiers and Miss H M Tönsing, who wrote the computer programs for the examples presented here.

Finally, I should like to thank Mrs M Russouw for her excellent typing of the manuscript.

Yaakov Yavin

Pretoria, June 1984

CONTENTS

CHAPTER 1 : PRELIMINARIES

1.1	INTRODUCTION	1
1.2	THE FUJISAKI-KALLIANPUR-KUNITA FILTERING FORMULA	8
1.3	THE 'STANDARD' NONLINEAR FILTERING PROBLEM	11
1.4	REFERENCES	24

CHAPTER 2 : ESTIMATION OF PARAMETERS VIA STATE OBSERVATION

2.1	INTRODUCTION	28
2.2	DERIVATION OF THE FILTER	29
2.3	AN ALGORITHM FOR COMPUTING $\hat{\eta}_x$	31
2.4	EXAMPLE 2.1 : SINE-WAVE OSCILLATOR	33
2.5	EXAMPLE 2.2 : TRIANGULAR-WAVES GENERATOR	35
2.6	ESTIMATION OF A MARKOV CHAIN	36
2.7	THE EQUATIONS OF OPTIMAL FILTERING	38
2.8	EXAMPLE 2.3 : POISSON PROCESS	41
2.9	EXAMPLE 2.4 : RANDOM TELEGRAPH SIGNAL	50
2.10	REMARKS	58
2.11	REFERENCES	59

CHAPTER 3 : FILTERING VIA MARKOV CHAINS APPROXIMATION

3.1	INTRODUCTION	60
3.2	CONSTRUCTION OF THE MARKOV CHAIN	62
3.3	THE EQUATIONS OF THE OPTIMAL FILTER	64
3.4	AN ALGORITHM FOR COMPUTING $\hat{\zeta}_x^{h,y}$	67
3.5	EXAMPLES : THE CASE $m=1$	70
3.6	EXAMPLES : THE CASE $m=2$	80
3.7	PARTIALLY OBSERVABLE SYSTEMS	90
3.8	REMARKS	98
3.9	REFERENCES	100

CHAPTER 4 : A KALMAN FILTER FOR A CLASS OF NONLINEAR STOCHASTIC
SYSTEMS

4.1	INTRODUCTION	103
4.2	THE DISCRETE-TIME MODEL	104
4.3	THE DISCRETE-TIME FILTER	107
4.4	EXAMPLE 4.1 : FREQUENCY PERTURBED SINE-WAVE OSCILLATOR	108
4.5	EXAMPLE 4.2 : A THREE PHASE SINE-WAVES GENERATOR	113
4.6	ESTIMATION WITH UNCERTAIN OBSERVATIONS	117
4.7	REMARKS	126
4.8	REFERENCES	128

CHAPTER 5 : APPROXIMATING FILTERS FOR CONTINUOUS-TIME SYSTEMS
WITH INTERRUPTED OBSERVATIONS

5.1	INTRODUCTION	130
5.2	CONSTRUCTION OF THE MARKOV CHAIN	132
5.3	THE EQUATIONS OF THE OPTIMAL FILTER	135
5.4	AN ALGORITHM FOR COMPUTING $(\hat{c}_x^{h,y}, \hat{\theta}^{h,y})$	138
5.5	EXAMPLES : THE CASE $m=1$	140
5.6	EXAMPLES : THE CASE $m=2$	148
5.7	REMARKS	165
5.8	REFERENCES	166

CHAPTER 6 : ESTIMATION IN A MULTITARGET ENVIRONMENT

6.1	INTRODUCTION	168
6.2	THE EQUATIONS OF THE OPTIMAL FILTER	169
6.3	AN ALGORITHM FOR COMPUTING $(\hat{\theta}, \hat{n})$	174
6.4	EXAMPLES	175
6.5	REFERENCES	181

CHAPTER 7 : STATE AND PARAMETER ESTIMATION

7.1	INTRODUCTION	182
7.2	CONSTRUCTION OF THE MARKOV CHAIN	184
7.3	THE EQUATIONS OF THE OPTIMAL FILTER	186
7.4	AN ALGORITHM FOR COMPUTING $(\hat{\zeta}_X^{h,y}, \hat{\theta}^{h_1,y})$	190
7.5	EXAMPLES : THE CASE $m=1$	191
7.6	EXAMPLES : THE CASE $m=2$	206
7.7	REMARKS	214
7.8	REFERENCES	214

CHAPTER 8 : STATE ESTIMATION FOR SYSTEMS DRIVEN BY WIENER
AND POISSON PROCESSES

8.1	INTRODUCTION	215
8.2	CONSTRUCTION OF THE MARKOV CHAIN	216
8.3	THE EQUATIONS OF THE OPTIMAL FILTER	219
8.4	EXAMPLES : THE CASE OF $m=1$	222
8.5	EXAMPLES : THE CASE $m=2$	228
8.6	AN EXTENSION OF EQUATION (8.1)	236
8.7	REFERENCES	238

CHAPTER 9 : PREDICTION VIA MARKOV CHAINS APPROXIMATION

9.1	INTRODUCTION	240
9.2	THE EQUATIONS OF OPTIMAL PREDICTION	241
9.3	AN ALGORITHM FOR COMPUTING $\hat{\zeta}_X^{h,y}(t,s)$	244
9.4	EXAMPLES	246

CHAPTER 10: SOME EXTENSIONS OF LINEAR FILTERING

10.1	LINEAR FILTERING WITH NON-GAUSSIAN INITIAL CONDITIONS	254
10.2	ESTIMATION OF MANEUVERING TARGETS	262
10.3	A DETECTION PROBLEM	267

10.4	STATE AND COVARIANCE ESTIMATION	270
10.5	CONCLUSIONS	272
10.6	REFERENCES	273