

# Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

1072

---

Franz Rothe

Global Solutions of  
Reaction-Diffusion Systems

---



Springer-Verlag  
Berlin Heidelberg New York Tokyo 1984

**Author**

Franz Rothe  
Lehrstuhl für Biomathematik, Universität Tübingen  
Auf der Morgenstelle 28, 7400 Tübingen, Federal Republic of Germany

AMS Subject Classifications (1980): 35B35, 35B40, 35B45, 35B65,  
35K55, 92A17, 92A40

ISBN 3-540-13365-8 Springer-Verlag Berlin Heidelberg New York Tokyo  
ISBN 0-387-13365-8 Springer-Verlag New York Heidelberg Berlin Tokyo

Library of Congress Cataloging in Publication Data Rothe, Franz, 1947 – Global solutions of reaction-diffusion systems. (Lecture notes in mathematics; 1072) Bibliography: p. Includes index. 1. Differential equations, Partial-Numerical solutions. 2. Differential equations, Parabolic-Numerical solutions. 3. Biomathematics. I. Title. II. Series: Lecture notes in mathematics (Springer-Verlag); 1072. QA3.L28 no. 1072 [QA377] 515.3'53 84-13887 ISBN 0-387-13365-8 (U.S.)

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, re-use of illustrations, broadcasting, reproduction by photocopying machine or similar means, and storage in data banks. Under § 54 of the German Copyright Law where copies are made for other than private use, a fee is payable to "Verwertungsgesellschaft Wort", Munich.

© by Springer-Verlag Berlin Heidelberg 1984  
Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach / Bergstr.  
2146 / 3140-543210

## Preface

This monograph is motivated by some problems from Mathematical Biology. Although there exists an extensive literature about nonlinear parabolic differential equations, none of the known results could be used to prove global existence of solutions for the reaction-diffusion systems considered in this monograph. In this situation, I gave an ad hoc proof of global existence for the one-dimensional reaction-diffusion system with reaction  $A + B \rightleftharpoons C$  subject to the mass action law. Afterwards it turned out that the method used could be generalized and applied to other problems as well. For the time being, the subject is not yet exhausted. Further interesting examples from applications are needed in order to build a substantial theory which is not just an unnecessarily abstract disguise of some specific problem. The author hopes that this monograph will be useful to stimulate research in this direction.

I take the opportunity to thank Prof. Dr. K.-P. Hadeler from the Lehrstuhl of Biomathematik, University of Tübingen, for his intensive personal and scientific support as well as my colleagues Dr. W. Ebel and Dr. H. Munz for reading the manuscript and giving much constructive criticism. With this help, I hope, the manuscript became more rigorous and more readable at the same time.

May 1984

Franz Rothe

## Contents

Introduction.....	1
Part I	
<u>Existence and A Priori Estimates for</u> <u>Reaction-Diffusion Equations</u> .....	5
Basic Notations and Definitions.....	11
Theorem 1 (Existence of mild solutions).....	32
Theorem 2 (Existence of mild solutions in the case of minimal regularity of the initial data) .....	33
Corollary of Theorem 1 (Uniqueness and maximality).....	54
Theorem 3 (Existence results exploiting a priori estimates).....	57
Theorem 4 (Global existence and global a priori estimates).....	67
Theorem 5 (Results on the behavior of the solution at a finite maximal existence time, which are available without global Lipschitz condition) ..	76
Theorem 6 (Global existence and uniform a priori estimates in the case without global Lipschitz condition).....	91
Theorem 7 (Stronger results for sublinear equations using only weak primary a priori estimates) .....	102
Part II	
<u>Some Examples of Reaction-Diffusion Systems</u> <u>Arising in Applications</u> .....	104
Review of Standard Theorems.....	108
Theorem 1 (Existence of mild and classical solutions).....	111
Theorem 2 (Construction of global solutions for irregular initial data) .....	120
Theorem 3 (Comparison of solutions by the strong maximum principle) .....	123
The Gierer-Meinhardt Model.....	126
Theorem (Globally bounded solutions for space dimension $N = 1, 2, 3$ ) .....	126

The Brusselator.....	140
Theorem 1 (Globally bounded solutions for space dimension $N = 1,2,3$ ) .....	140
Theorem 2 (Global solution for space dimension $N = 4$ ).....	146
The FitzHugh-Nagumo System.....	148
Theorem 1 (Global solutions for arbitrary space dimension).....	149
Theorem 2 (Sufficient conditions for decay of solutions in space dimension $N \leq 3$ ) .....	154
Chemical Reactions.....	157
Theorem 1 (Globally bounded solutions for space dimension $N \leq 5$ ) .....	157
Theorem 2 (Asymptotic behavior by means of entropy).....	167
A Nuclear Reactor Model.....	172
Theorem (Boundedness and convergence to equilibrium).....	173
The Volterra-Lotka Model.....	188
Theorem 1 (Boundedness and convergence to equilibrium).....	189
Theorem 2 (Degenerate cases with one nondiffusing species).....	190
Theorem 3 (Boundedness and convergence to equilibrium for some generalized Volterra-Lotka systems) .....	207
References.....	211
Index.....	215