

# Lecture Notes in Mathematics

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## Function Spaces and Applications

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## F o r e w o r d

These are the proceedings of a seminar held in Lund in June 1986, devoted to function space methods in analysis. The organizers felt that recent developments in Interpolation Theory have important implications in various areas of analysis, and that further work in Interpolation would benefit from the input of new problems and applications. Thus the makeup of the seminar was considerably broader than its two predecessors (Lund 1982 and 1983). It is particularly appropriate that these seminars have taken place in Lund, where Interpolation Theory was born some 60 years ago. To further emphasize the historical origin of the subject, the collection is preceded by a historical lecture on the life of Marcel Riesz.

The organizers wish to thank Naturvetenskapliga Forskningsrådet and the N.S.F. for supporting the conference. The two non-Swedish organizers feel that they express the sentiments of all participants in thanking Jaak Peetre and Hans Wallin as well as the members of the mathematics department at Lund for their warm hospitality.

The Editors

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