

# Lecture Notes in Control and Information Sciences

Edited by M. Thoma and A. Wyner

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J. S. Freudenberg, D. P. Looze

Frequency Domain Properties  
of Scalar and Multivariable  
Feedback Systems

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To my grandparents,  
J.S. Freudenberg

To my wife,  
D.P. Looze

## **Preface**

The research reported in this monograph was initiated when both authors were at the University of Illinois; and continued when the second author moved to ALPHATECH and the University of Massachusetts and the first author moved to the University of Michigan. Although our research program is still in development, we feel that enough has been accomplished to warrant a comprehensive presentation of our approach.

We would like to acknowledge helpful discussions with many colleagues. In particular, the first author is grateful for the opportunity to interact with the personnel of the Honeywell Systems and Research Center during the summer of 1982. Discussions with Bruce Francis and George Zames have also proven quite useful. An enormous debt of gratitude is owed to Professor Stephanie Alexander, of the Mathematics Department at the University of Illinois. Without her enthusiastic assistance, the work reported in Chapters 8-11 would never have been completed. The word-processing is due to Elizabeth Olsen and Virginia Folsom of the University of Michigan. Finally, the work of the first author was supported in part by the United States National Science Foundation, under Grant ECS 8504558.

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