# Lecture Notes in Mathematics

## 883

# Cylindric Set Algebras

Cylindric Set Algebras and Related Structures
By L. Henkin, J. D. Monk, and A. Tarski

On Cylindric-Relativized Set Algebras
By H. Andréka and I. Németi

**Authors**

Leon Henkin
Department of Mathematics, University of California
Berkeley, CA 94720, USA

J. Donald Monk
Department of Mathematics, University of Colorado
Boulder, CO 80309, USA

Alfred Tarski
462 Michigan Ave.
Berkeley, CA 94707, USA

Hajnalka Andréka
István Németi
Mathematical Institute, Hungarian Academy of Sciences
Reáltanoda u. 13–15, 1053 Budapest, Hungary

## Introduction

This volume is devoted to a comprehensive treatment of certain set-theoretical structures which consist of fields of sets enhanced by additional fundamental operations and distinguished elements. The treatment is largely self-contained.

Each of these structures has an associated dimension $\alpha$, a finite or infinite ordinal; their basic form is well illustrated in the case $\alpha = 3$. Let $R$ be an arbitrary set, and let $\mathscr{F}$ be a field of subsets of the set $^3R$ of all triples of elements of $R$. Thus $\mathscr{F}$ is a non-empty collection of subsets of $^3R$ closed under union, intersection, and complementation relative to $^3R$. We shall assume that $\mathscr{F}$ is closed under the three operations $C_0, C_1, C_2$ of <u>cylindrification</u>, where $C_0$, for example, is the operation given by:

$$C_0 X = \{\langle x,y,z \rangle : \langle u,y,z \rangle \in X \text{ for some } u, \text{ with } x \in R\} ;$$

$C_0 X$ is the cylinder formed by moving $X$ parallel to the first axis. $C_1 X$ and $C_2 X$ are similarly related to the second and third axes. We also assume that the <u>diagonal planes</u> $D_{01}, D_{02}, D_{12}$ are in $\mathscr{F}$; here, for example,

$$D_{01} = \{\langle x,x,y \rangle : x,y \in R\} .$$

Similarly $D_{02}$ (resp., $D_{12}$) consists of all triples of $^3R$ whose first and third (resp., second and third) coordinates coincide. A collection $\mathscr{F}$ satisfying all of these conditions is called a <u>cylindric field of sets</u> (of dimension 3). Cylindric fields of sets and certain closely related structures are the objects of study in this volume. Considered not merely as collections of sets, but as algebraic objects endowed with fundamental

operations and distinguished elements, cylindric fields of sets are called
cylindric set algebras. $Cs_\alpha$ is the class of all cylindric set algebras
of dimension $\alpha$ , and $ICs_\alpha$ is the class of algebras isomorphic to them.

In much of the work, general algebraic notions are studied in their
application to cylindric set algebras. We consider subalgebras, homomorphisms,
products, and ultraproducts of them, paying special attention, for example,
to the closure of $ICs_\alpha$ and related classes under these operations. In
addition, there are natural operations upon these structures which are
specific to their form as certain Boolean algebras with operators, such as
relativization to subsets of $^3R$ and isomorphism to algebras of subsets of
$^3S$ with $S \neq R$ , and there are relationships between set algebras of
different dimensions.

Although, as mentioned, the volume is largely self-contained, we shall
often refer to the book Cylindric Algebras, Part I, by Henkin, Monk, and
Tarski. Many notions touched on briefly in the present volume are treated
in detail in that one, and motivation for considering certain questions can
be found there. Indeed, the present work had its genesis in the decision
by Henkin, Monk, and Tarski to publish a series of papers which would form
the bulk of Part II of their earlier work. Their contribution to the present
volume is, in fact, the first of this proposed series. As their writing
proceeded, they learned of the closely related results obtained by Andréka
and Németi, and invited the latter to publish jointly with themselves.

Thus, the present volume consists of two parts. The first, by Henkin,
Monk, and Tarski, contains the basic defintions and results on various kinds of
cylindric set algebras. The second, by Andréka and Németi, is organized
parallel to the first. In it, certain aspects of the theory are investigated
more thoroughly; in particular, many results which are merely formulated

in Part I, are provided with proofs in Part II.  In both parts, many
open problems concerning the structures considered are presented.


The authors

Berkeley
Boulder
Budapest

# Table of contents