

# Lecture Notes in Mathematics

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## Kinetic Theories and the Boltzmann Equation

Lectures given at the 1<sup>st</sup> 1981 Session of the  
Centro Internazionale Matematico Estivo (C.I.M.E.)  
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## PREFACE

The book contains the text of three of the four series of lectures plus a few seminars presented at the first session of the Summer School organized at C.I.M.E. We regret very much that we were unable to obtain the text of the lectures on the singular limits of the Boltzmann Equation by Professor Harold Grad, in spite of the fact that we waited for them more than two years.

The unifying theme of the School was the study of evolution equations whose unknown is a distribution function describing the probabilistic behaviour of the underlying particle dynamics.

The lecture notes presented in this volume deal with the time dependent linear transport equation (by Professor J.J. Hejtmanek of Vienna University), the existence theorems for the nonlinear Boltzmann equation (by Professor P.F. Zweifel of Virginia Polytechnic Institute and State University), the Boltzmann-Vlasov equation for ionized gases (by Professor H. Neunzert of the University of Kaiserslautern).

The text of the seminars deals with half space problems for kinetic models, the Boltzmann equation for molecular forces of infinite range, a survey of recent results of the Cauchy problem for the Boltzmann equation, the Boltzmann hierarchy.

Except for the forced omission mentioned above, we feel that the volume gives a coherent picture of this field of applied mathematics.

Carlo Cercignani

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