Lecture Notes in Mathematics

Edited by A. Dold and B. Eckmann

Subseries: Instituto de Matemática Pura e Aplicada, Rio de Janeiro
Adviser: C. Camacho

1050

Alexander Prestel
Peter Roquette

Formally p-adic Fields

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

Authors

Alexander Prestel
Fakultät für Mathematik, Universität Konstanz
Postfach 5560, 7750 Konstanz, Federal Republic of Germany

Peter Roquette
Mathematisches Institut, Universität Heidelberg
Im Neuenheimer Feld 288, 6900 Heidelberg, Federal Republic of Germany

This book is also available as no. 38 of the series "Monografias de Matemática", published by the Instituto de Matemática Pura e Aplicada, Rio de Janeiro

AMS Subject Classifications (1980): 12B99, 12J10

ISBN 3-540-12890-5 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-12890-5 Springer-Verlag New York Heidelberg Berlin Tokyo

Printed in Germany

Printing and binding: Beltz Offsetdruck, Hemsbach/Bergstr.
2146/3140-543210

Preface

These notes result from lectures given by the authors at IMPA in Rio de Janeiro - in 1980 by the second and in 1982 by the first author. The 1980 course was mainly concerned with the content of Sections 6 and 7 , using as a prerequesite the Ax-Kochen-Eršov- Theorem on the model completeness of the theory of p-adically closed fields. After that, an algebraic approach to this important theorem as well as an analysis of p-adic closures was developed (contained in Sections 3 and 4). The present lecture notes essentially coincide with the 1982 course.

In the introductory Section 1 we try to point out the analogy between the theory of p-valued fields and the well-known theory of ordered fields. After giving some basic definitions and examples as well as basic facts from general valuation theory in Section 2, we develop the theory of p-valued fields, i.e. fields together with a fixed p-valuation in Section 3 and 4 . From Section 6 on we no longer fix a certain p-valuation, instead we only assume the existence of such a valuation. Fields which admit some p-valuation are called formally p-adic. The theory of formally p-adic fields is concerned with the investigation of all p-valuations rather than just one. In Section 7 we concentrate on the important case of function fields. In Section 5 we use results proved in previous sections for model theoretic investigations of formally p-adic fields. In particular we deduce the Ax-Kochen-Eršov-Theorem. Viewed historically, these results stand at the beginning of the development of formally p-adic fields.

The only Section which makes use of model theoretic notions and facts is Section 5 . However, there is one exception - the notion of saturated structures - which is used also in the formulation and proof of the Embedding Theorem in Section 4 . We consider this notion as very useful in the investigation of function fields, since with its help 'specialization theorems' become essentially equivalent to 'embedding theorems'. Actually, it is this equivalence which is used in Section 7 , Theorem 7.2 . In order to keep this book as selfcontained as possible, we add in Proposition 7.3 elementary proofs of all the facts needed here about saturated fields. The reader who is interested in the general theory of saturated structures is referred to the books [B-S],[C-K], and [S].

The authors wish to thank all colleagues who offered helpful suggestions in the preparation of these notes, and owe a special debt to F.V. Kuhlmann for reading the complete manuscript and setting up the notation and the subject index. Last but not least we are grateful to Edda Polte for preparing the typescript.

Heidelberg-Konstanz 1983

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