

Lecture Notes in Mathematics

1480

Editors:

A. Dold, Heidelberg

B. Eckmann, Zürich

F. Takens, Groningen

Subseries:

Instituto de Matemática Pura e Aplicada

Rio de Janeiro, Brazil (vol. 48)

Adviser:

C. Camacho



F. Dumortier R. Roussarie
J. Sotomayor H. Żoładek

Bifurcations of Planar Vector Fields

Nilpotent Singularities and Abelian Integrals

Springer-Verlag

Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona
Budapest

Authors

Freddy Dumortier
Limburgs Universitaire Centrum
Universitaire Campus
3610 Diepenbeek, Belgium

Robert Roussarie
Département de Mathématiques
Université de Bourgogne
UFR de Sciences et Techniques
Laboratoire de Topologie
(U. A. no. 755 du CNRS), B. P. 138
21004 Dijon, France

Jorge Sotomayor
Instituto de Matemática Pura e Aplicada
Estrada Dona Castorina 110
CEP 22460 Jardim Botânico
Rio de Janeiro, Brazil

Henryk Żoładek
Institute of Mathematics
Warsaw University
00-901 Warsaw, Poland

Mathematics Subject Classification (1991): 58F14, 34C05, 34D30

ISBN 3-540-54521-2 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-54521-2 Springer-Verlag New York Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, re-use of illustrations, recitation, broadcasting, reproduction on microfilms or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

© Springer-Verlag Berlin Heidelberg 1991
Printed in Germany

Typesetting: Camera ready by author
Printing and binding: Druckhaus Beltz, Hemsbach/Bergstr.
46/3140-543210 - Printed on acid-free paper

PREFACE

The study of bifurcations of families of dynamical systems defined by vector fields (i.e. ordinary differential equations) depending on real parameters is at present an active area of theoretical and applied research.

Problems in mathematical biology, fluid dynamics, electrical engineering, among other applied disciplines, lead to multiparametric vector fields whose bifurcation analysis of equilibria (singular points) and oscillations (cycles) is required.

The case of planar vector fields, due to the presence of regular as well as singular limit cycles is the first one, in increasing dimension of phase space, whose study cannot be fully reduced to the analysis of singularities and zeroes of algebraic equations, particularly when the number of parameters involved is larger than or equal to two.

The results established in this volume illustrate the diversity of the algebraic, geometric and analytic methods used in the description of the variety of structural patterns that appear in the bifurcation diagrams of generic three-parameter families of planar vector fields, around singular points whose linear parts are nilpotent.

The analysis involved in their proofs and in the discussion of the remaining conjectures points out to the actual limits of established tools for the study of complex bifurcation problems.

The introductions to the two works which constitute this volume locate precisely, in the context of the current literature, the specific character of each of their contributions.

The authors

**Generic 3-Parameter Families of Planar Vector Fields,
Unfoldings of Saddle, Focus and Elliptic Singularities
With Nilpotent Linear Parts**

by

F. Dumortier, R. Roussarie, J. Sotomayor

Contents of the volume

Generic 3-parameter families of planar vector fields, unfoldings of saddle, focus and elliptic singularities with nilpotent linear parts.

by F. Dumortier, R. Roussarie, J. Sotomayor	VI
Table of Contents	VII
Part I Presentation of the Results and Normalization	1
Part II Rescalings an Analytic Treatment	57

Abelian integrals in unfoldings of codimension 3 singular planar vector fields.

by H. Zoladek	165
Table of Contents	166
Part I The weakened 16-th Hilbert Problem	167
Part II The Saddle and Elliptic Cases	173
Part III The Focus Case	193
Index	225

Table of contents

PART I: PRESENTATION OF THE RESULTS AND NORMALIZATION

Chapter I: Introduction	1
I.1: Position of the problem and statement of results	1
I.2: Codimension 1-phenomena	9
I.3: Conic structure of the bifurcation set and rescaling	13
I.4: Organization of the paper	15
Chapter II: Definitions and notations	19
Chapter III: Transformation into normal form	22
Chapter IV: Bifurcations of codimension 1 and 2	28
IV.1: Generalities	28
IV.2: Codimension 1 bifurcations	31
IV.3: Codimension 2 bifurcations	37

PART II: RESCALINGS AND ANALYTIC TREATMENT

Chapter V: Elementary properties	57
V.A: Location and nature of critical points	57
V.B: Location of the Hopf bifurcations of codimensions 1 and 2	59
V.C: Bifurcations along the set SN	66
V.D: Rotational property with respect to the parameter ν	71
V.E: The principal rescaling	83
Chapter VI: The central rescaling	85
VI.A: Definition and basic properties	85
VI.B: The saddle case	87
1. Hopf bifurcations	88
2. Integrating factor and Abelian integral	88
3. Saddle connections	94

4. Bifurcation point of two saddle connections	99
5. Complete analysis of the saddle case in a large central rescaling chart . .	102
6. Study in some “principal rescaling cone” around the TSC-line	114
VI.C: The elliptic case	117
VI.D: The focus case	120
1. Study along the μ_2 -axis in a large central rescaling chart	120
2. Study along the ν -axis	125
3. Study in a “principal rescaling cone” around the DH-line and the DL-line	133
Chapter VII: Conclusions and discussion of remaining problems	135
VII.A: The genral conjecture	135
VII.B: The saddle case	137
VII.C: The focus case	143
VII.D: The elliptic case	150
References	162