Lecture Notes in Mathematics

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PREFACE

These notes reflect a course on vector measures given at The University of Texas at Austin in the summer term of 1970. They contain as much material as is usual for a course on this topic.

The presentation is unusual, though, and might possibly be of some interest even to the expert. It is motivated by the desire to carry M. H. Stone's unified treatment of set functions and Radon measures as far as possible.

This has been achieved by the use of a new and, I believe, natural definition of measurability, far-reaching enough to embrace the definitions both of Bourbaki and Caratheodory-Halmos and combining the flexibility and intuitive appeal of the former with the applicability to probability theory of the latter. It can also be applied to more general situations than either of these.

Another unorthodox feature is the axiomatic treatment of upper integrals and their generalizations, the upper gauges, leading to the Daniell-integral also for measures and linear maps that do not have finite variation.

I should like to express my gratitude to Professor Roy P. Kerr, who eradicated a frightful number of mistakes and Germanisms from the original manuscript, and to NSF for their support of these notes; and last but not least to Ms. Linda S. White who typed these notes with extreme skill and diligence and with inexhaustible patience.

Klaus Bichteler *

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